MATH 221–02 (Kunkle), Quiz 4 Name: ______________________

10 pts, 10 minutes Feb 15, 2024

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1 (10 pts). Let $g(x,y) = \dfrac{x}{2x-y}$. Find g_x, g_y, g_{xx}, g_{xy}, and g_{yy}.
Express your answers either as products (using negative exponents), or as quotients in lowest terms. Perform basic simplifications.

Solution:

1.(Source: 14.3.22,23,55) Here a solution using the product and chain rules. It pays to factor out the lowest power of $2x-y$ throughout.

$$\begin{aligned} g &= x(2x-y)^{-1} \\ g_x &= (2x-y)^{-1} + x(-1)(2x-y)^{-2}2 \\ &= (2x-y-2x)(2x-y)^{-2} \\ &= -y(2x-y)^{-2} \\ g_y &= x(2x-y)^{-2} \end{aligned}$$

$$\begin{aligned} g_{xx} &= 4y(2x-y)^{-3} \\ g_{xy} &= -(2x-y)^{-2} - y(-2)(2x-y)^{-3}(-1) \\ &= -(2x-y)^{-3}(2x-y+2y) \\ &= -(2x-y)^{-3}(2x+y) \\ g_{yy} &= 2x(2x-y)^{-3} \end{aligned}$$

If you used the quotient and chain rules, your answers should look like this:

$$\begin{aligned} g_x &= \frac{1(2x-y) - x\cdot 2}{(2x-y)^2} = \frac{-y}{(2x-y)^2} \\ g_y &= \frac{0(2x-y) - x\cdot(-1)}{(2x-y)^2} = \frac{x}{(2x-y)^2} \\ g_{xx} &= \frac{0(2x-y)^2 + y2(2x-y)2}{(2x-y)^4} = \frac{4y}{(2x-y)^3} \\ g_{xy} &= \frac{-1(2x-y)^2 + y2(2x-y)(-1)}{(2x-y)^4} = \frac{-1(2x-y) - 2y}{(2x-y)^3} = \frac{-(2x+y)}{(2x-y)^3} \\ g_{yy} &= \frac{0(2x-y)^2 - x\cdot 2(2x-y)(-1)}{(2x-y)^4} = \frac{2x}{(2x-y)^3} \end{aligned}$$