
1 (10 pts). In each part, find the point (x, y, z) of intersection or show that none exists.

a. The point of intersection of the lines

$$\begin{array}{lll} x = 2 & y = t - 5 & z = -3 + 2t \\ x = 5 - s & y = -15 + 4s & z = 7 - 2s \end{array}$$

b. The point of intersection of the lines

$$\begin{array}{lll} x = 2 & y = t - 5 & z = -3 + 2t \\ x = 5 - s & y = -15 + 4s & z = 8 - 2s \end{array}$$

Solution:

1a.(Source: 12.5.21) Set the coordinates equal and solve for s and t :

$$(1) \quad \begin{array}{lll} 2 = 5 - s & & s = 3 \\ t - 5 = -15 + 4s & \Rightarrow & 4s - t = 10 \\ -3 + 2t = 7 - 2s & & 2s + 2t = 10 \end{array}$$

The 1st and 2nd equations yield $t = 2$, $s = 3$, and when we check, these also satisfy the 3rd. Therefore, the solution to (1) is $s = 3$, $t = 2$. Substituting these into the equation of either line gives the intersection point $(x, y, z) = (2, -3, 1)$.

1b.(Source: 12.5.19) Likewise, solve

$$(2) \quad \begin{array}{lll} 2 = 5 - s & & s = 3 \\ t - 5 = -15 + 4s & \Rightarrow & 4s - t = 10 \\ -3 + 2t = 8 - 2s & & 2s + 2t = 11 \end{array}$$

The 1st and 2nd equations again yield $t = 2$, $s = 3$, but when we check the third,

$$2 \cdot 3 + 2 \cdot 2 = 10 \neq 11,$$

and so (2) has no solution and lines don't intersect.