1. Suppose $f(x, y)$ is a differentiable function of $x$ and $y$.

<table>
<thead>
<tr>
<th>$(x, y)$</th>
<th>$f$</th>
<th>$f_x$</th>
<th>$f_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(0, 0)$</td>
<td>1</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>$(0, 1)$</td>
<td>−5</td>
<td>2</td>
<td>−6</td>
</tr>
</tbody>
</table>

a (3 pts). Find the derivative of $f$ at $(x, y) = (0, 0)$ in the direction $\langle \frac{5}{13}, -\frac{12}{13} \rangle$.

b (3 pts). At $(x, y) = (0, 0)$, what is the greatest directional derivative of $f$, and in what direction (unit vector) does it occur?

c (4 pts). Suppose $z = f(9u - 7v, 1 - 8u)$. Find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ at $(u, v) = (0, 0)$.

**Solution:**

1a. (Source: 14.6.7-8, 11-12)

$$D_{\langle \frac{5}{13}, -\frac{12}{13} \rangle} f(0, 0) = \nabla f(0, 0) \cdot \langle \frac{5}{13}, -\frac{12}{13} \rangle$$

$$= \langle 3, 4 \rangle \cdot \langle \frac{5}{13}, -\frac{12}{13} \rangle$$

$$= \frac{15}{13} - \frac{48}{13} = -\frac{33}{13}.$$ 

1b. (Source: 14.6.21-23) The greatest value of of the directional derivative of $f$ at $(0, 0)$ is $|\nabla f| = |\langle 3, 4 \rangle| = 5$.

It occurs in the direction

$$u = \frac{1}{|\nabla f|} \nabla f = \langle \frac{3}{5}, \frac{4}{5} \rangle.$$ 

1c. (Source: 14.5.15-16) $(x, y) = (0, 1)$ when $(u, v) = (0, 0)$. At that point,

$$\frac{\partial z}{\partial u} = f_x x_u + f_y y_u$$

$$= 2(9) - 6(-8) = 66.$$ 

$$\frac{\partial z}{\partial v} = f_x x_v + f_y y_v$$

$$= 2(-7) - 6(0) = -14.$$