1. Let \( g(x, y) = \frac{x}{2x - y} \). Find \( g_x, g_y, g_{xx}, g_{xy}, \) and \( g_{yy} \).

Express your answers either as products (using negative exponents), or as quotients in lowest terms. Perform basic simplifications.

**Solution:**

1. (Source: 14.3.22,23,55) Here a solution using the product and chain rules. It pays to factor out the lowest power of \( 2x - y \) throughout.

\[
\begin{align*}
g &= x(2x - y)^{-1} \\
g_x &= (2x - y)^{-1} + x(-1)(2x - y)^{-2} \\
&= (2x - y - 2x)(2x - y)^{-2} \\
&= -y(2x - y)^{-2} \\
g_y &= x(2x - y)^{-2} \\
g_{xx} &= 4y(2x - y)^{-3} \\
g_{xy} &= -(2x - y)^{-2} - y(-2)(2x - y)^{-3}(-1) \\
&= -(2x - y)^{-3}(2x - y + 2y) \\
&= -(2x - y)^{-3}(2x + y) \\
g_{yy} &= 2x(2x - y)^{-3}
\end{align*}
\]

If you used the quotient and chain rules, your answers should look like this:

\[
\begin{align*}
g_x &= \frac{1(2x - y) - x \cdot 2}{(2x - y)^2} = \frac{-y}{(2x - y)^2} \\
g_y &= \frac{0(2x - y) - x \cdot (-1)}{(2x - y)^2} = \frac{x}{(2x - y)^2} \\
g_{xx} &= \frac{0(2x - y)^2 + y2(2x - y)2}{(2x - y)^4} = \frac{4y}{(2x - y)^3} \\
g_{xy} &= \frac{-1(2x - y)^2 + y2(2x - y)(-1)}{(2x - y)^4} = \frac{-1(2x - y) - 2y}{(2x - y)^3} = \frac{-(2x + y)}{(2x - y)^3} \\
g_{yy} &= \frac{0(2x - y)^2 - x \cdot 2(2x - y) - 1}{(2x - y)^4} = \frac{2x}{(2x - y)^3}
\end{align*}
\]