1 (10 pts). Let \( \mathbf{u} = \langle 2, -3, 1 \rangle \) and \( \mathbf{v} = \langle 1, 2, -1 \rangle \).

a. Find the vector projection of \( \mathbf{v} \) onto \( \mathbf{u} \).

b. Find the scalar projection of \( \mathbf{v} \) onto \( \mathbf{u} \).

c. Find the cosine of the angle between \( \mathbf{u} \) and \( \mathbf{v} \). Is the angle acute or obtuse?

**Solution:**

1a. (Source: 12.3.42)

\[
\text{proj}_u \mathbf{v} = \frac{\mathbf{v} \cdot \mathbf{u}}{\mathbf{u} \cdot \mathbf{u}} \mathbf{u} = \frac{2 \cdot 1 + (-3) \cdot 2 + 1 \cdot (-1)}{2^2 + (-3)^2 + 1^2} \langle 2, -3, 1 \rangle = \frac{-5}{14} \langle 2, -3, 1 \rangle, \text{ or } \langle -\frac{5}{7}, \frac{15}{14}, -\frac{5}{14} \rangle.
\]

1b. (Source: 12.3.42)

\[
\text{comp}_u \mathbf{v} = \frac{\mathbf{v} \cdot \mathbf{u}}{|\mathbf{u}|} = \frac{-5}{\sqrt{14}}.
\]

The scalar projection is the same as \( |\mathbf{v}| \cos \theta \), the signed length of the vector projection. In this case, the sign is negative, because the projection onto \( \mathbf{u} \) points in the opposite direction as \( \mathbf{u} \).

1c. (Source: 12.3.17) The lengths of \( \mathbf{u} \) and \( \mathbf{v} \) are

\[
|\mathbf{u}| = \sqrt{\mathbf{u} \cdot \mathbf{u}} = \sqrt{14}
\]
\[
|\mathbf{v}| = \sqrt{\mathbf{v} \cdot \mathbf{v}} = \sqrt{6}
\]

The cosine of the angle between \( \mathbf{u} \) and \( \mathbf{v} \) is \( \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}| |\mathbf{v}|} = \frac{-5}{\sqrt{14} \sqrt{6}} \), or \( \frac{-5}{2 \sqrt{21}} \). Since the cosine is negative, the angle is **obtuse**.