No notes, books, electronic devices, or outside materials of any kind.

Read each problem carefully and simplify your answers.

Supporting work will be required on every problem worth more than 2 points unless instructions say otherwise.

You are expected to know the values of all trig functions at multiples of $\pi/4$ and of $\pi/6$.

1(19 pts). Find the area of the part of the surface $x^2 - \sqrt{3}y + z = 15$ that lies above (or below) the triangle with vertices $(0,0)$, $(2,2)$, $(2,-2)$.

2(12 pts). Find the volume of the solid below $z = (x+y)^2$ and above the square in the xy-plane with vertices $(0,1)$, $(1,1)$, $(0,2)$, and $(1,2)$.

3(15 pts). Evaluate the double integral $\iint_R \frac{y}{x^2+y^2}\,dA$ where R is the region between the circles $x^2+y^2=9$ and $x^2+y^2=1$ and above the x-axis. That is, $R = \{\,(x,y) \bigm| 1 \le x^2+y^2 \le 9,\ 0 \le y\,\}$.

4(16 pts). Let T be the tetrahedron in the first octant bounded by the coordinate planes and the plane $4x+y+z=4$. Write the triple integral $\iiint_T 1\,dV$ as an iterated integral in the given order. Do not evaluate this integral.

a. $\iiint 1\,dz\,dy\,dx$

b. $\iiint 1\,dx\,dy\,dz$.

5. Let H be the solid above the cone $z = \sqrt{x^2+y^2}$ and below the plane $z=1$.

a(12 pts). Write the triple integral $\iiint_H (x^2+y^2)\,dV$ as an iterated integral in cylindrical coordinates. Do not evaluate this integral.

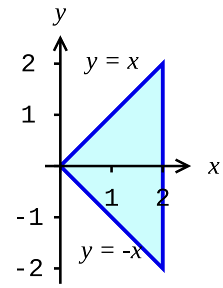
b(12 pts). Write the triple integral $\iiint_H (x^2+y^2)\,dV$ as an iterated integral in spherical coordinates. Do not evaluate this integral.

6(14 pts). Let P be the interior of the parallelogram enclosed by the lines

$$x+3y=0 \qquad x+3y=1 \qquad x-2y=1 \qquad x-2y=3$$

Rewrite the double integral $\iint_P y\,dA$ as an iterated integral in the variables $u = x+3y$ and $v = x-2y$. Do not evaluate this integral.

1(19 pts).(Source: 15.5.4) The area of the surface $z = 15 - x^2 + \sqrt{3}y$ is the integral of $dS = \sqrt{1 + z_x^2 + z_y^2}\,dx\,dy = \sqrt{1 + (2x)^2 + \sqrt{3}^2}\,dx\,dy = \sqrt{4 + 4x^2}\,dx\,dy$. This integral is easiest to compute in the order $dy\,dx$:



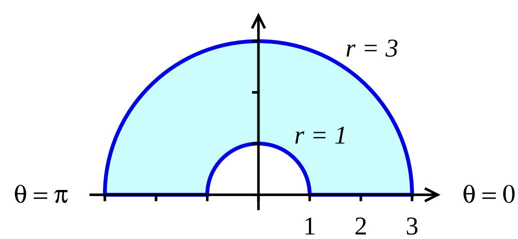
$$S = \int_0^2 \int_{-x}^{x} 2\sqrt{1 + x^2}\,dy\,dx$$
$$= \int_0^2 4x\sqrt{1 + x^2}\,dx = \frac{4}{3}(1 + x^2)^{3/2}\Big|_0^2 = \frac{4}{3}(5^{3/2} - 1).$$

2(12 pts).(Source: 15.1.43) The volume is the double integral $\int_0^1 \int_1^2 (x + y)^2\,dy\,dx$. It's easiest to integrate if we notice that $(x + y)_x$ and $(x + y)_y$ both equal 1 and *avoid* multiplying out powers of $(x + y)$:

$$\int_0^1 \int_1^2 (x + y)^2\,dy\,dx = \int_0^1 \frac{1}{3}(x + y)^3\Big|_1^2\,dx = \int_0^1 \frac{1}{3}\big((x + 2)^3 - (x + 1)^3\big)\,dx$$
$$= \frac{1}{12}\big((x + 2)^4 - (x + 1)^4\big)\Big|_0^1 = \frac{1}{12}(3^4 - 2^4 - 2^4 + 1) = \frac{25}{6}.$$

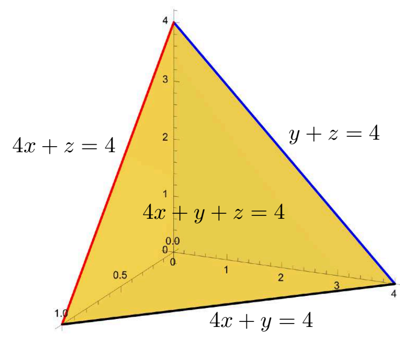
3(15 pts).(Source: 15.3.10,13) Convert to polar, using $x^2 + y^2 = r^2$ and $y = r\sin\theta$ and $dA = r\,dr\,d\theta$:

$$\int_0^{\pi} \int_1^3 \frac{r\sin\theta}{r^2} r\,dr\,d\theta = \int_0^{\pi} \int_1^3 \sin\theta\,dr\,d\theta$$
$$= \int_0^{\pi} 2\sin\theta\,d\theta = -\cos\theta\Big|_0^{\pi} = 2$$



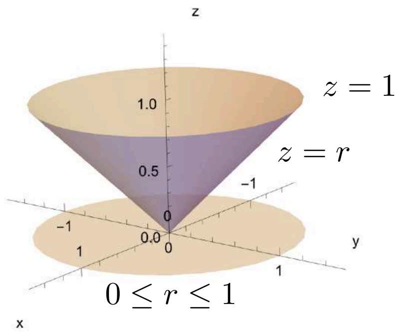
Several students assumed that the integral over R is twice the integral over the right half of R. That's true for this particular integrand (because $f(x, y)$ is even in x) but not in general, so I penalized anyone who made that assumption without some reason.

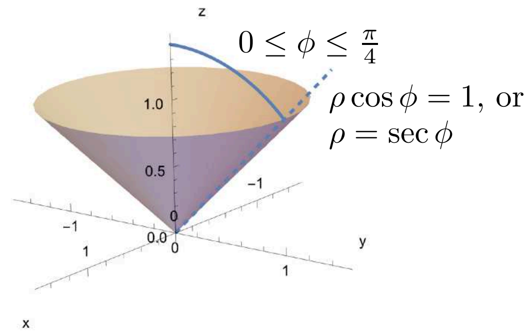
4(16 pts).(Source: 15.6.16,29-36) See the figure for the graph of $4x + y + z = 4$. To find the equations of the lines of intersection (shown in red, blue, and black) of the plane and the coordinate planes, substitute x, y, or $z = 0$.



a. $$\int_0^1 \int_0^{4-4x} \int_0^{4-y-4x} dz\,dy\,dx$$

b. $$ds \int_0^4 \int_0^{4-z} \int_0^{1-\frac{1}{4}y-\frac{1}{4}z} dx\,dy\,dz$$





5a(12 pts).(Source: 15.7.32) $x^2 + y^2 = r^2$ and $dV = r\, dz\, dr\, d\theta$. Integral is $\int_0^{2\pi} \int_0^1 \int_r^1 \, dz\, r^3\, dr\, d\theta = \int_0^{2\pi} \int_0^1 \int_0^z \, r^3\, dr\, dz\, d\theta$.

5b(12 pts).(Source: 15.8.15) $r^2 = \rho^2 \sin^2\phi$ and $dV = \rho^2 \sin\phi\, d\rho, d\phi\, d\theta$. Translate $z = 1$ to spherical coordinates using $z = \rho\cos\phi$. See figure. $\int_0^{2\pi} \int_0^{\pi/2} \int_0^{\sec\phi} \rho^4 \sin^3\phi\, dp\, d\phi\, d\theta$

6(14 pts).(Source: 15.9.23) The double integral equals $\displaystyle\int_0^1 \int_1^3 y \left| \frac{\partial(x,y)}{\partial(u,v)} \right| \, dv\, du$. Solve for u and v in terms of x and y:

$$\begin{array}{l} u = x + 3y \\ \underline{v = x - 2y} \\ u - v = 5y \end{array} \quad \Longrightarrow \quad \begin{array}{l} y = \frac{1}{5}u - \frac{1}{5}v \\ x = v + 2y = \frac{2}{5}u + \frac{3}{5}v \end{array}$$

Then

$$\left| \frac{\partial(x,y)}{\partial(u,v)} \right| = \left| \det \begin{pmatrix} x_u & x_v \\ y_u & y_v \end{pmatrix} \right| = \left| \det \begin{pmatrix} \frac{2}{5} & \frac{3}{5} \\ \frac{1}{5} & -\frac{1}{5} \end{pmatrix} \right| = \left| -\tfrac{2}{25} - \tfrac{3}{25} \right| = \tfrac{1}{5}$$

and the integral equals

$$\int_0^1 \int_1^3 (\tfrac{1}{5}u - \tfrac{1}{5}v)\tfrac{1}{5} \, dv\, du.$$