1. (5 pts) Find the center and radius of the sphere $x^2 - 2x + y^2 + z^2 + 4z = 4$.

2. (12 pts) Sketch the graph of the given equation in each part. Label your axes $x$, $y$, $z$ and use arrows to indicate the positive direction along each. Label your answers so I can tell which is which.
   a. $4x^2 = z$
   b. $4x^2 + y^2 = z$

3. (14 pts) Let $\mathbf{u} = -2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $\mathbf{v} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$. Find each of the following.
   a. The cosine of the angle between $\mathbf{u}$ and $\mathbf{v}$.
   b. The vector projection of $\mathbf{v}$ onto $\mathbf{u}$.
   c. A nonzero vector perpendicular to both $\mathbf{u}$ and $\mathbf{v}$.

4. (7 pts) Find an equation of the plane passing through the three points $(0, 1, 0)$, $(-2, 3, 1)$, $(1, 4, -2)$.

5. (5 pts) Find an equation for the line passing through the two points $(4, 0, 1)$ and $(2, 2, 2)$.

6. Let $\mathbf{r}(t) = <2t, t^2, \ln t>$ be the position of a particle at time $t$.
   a. (7 pts) Express the particle’s velocity, acceleration, and speed as functions of $t$. Label your answers so I can tell which is which.
   b. (8 pts) Find the distance traveled by the particle from time $t = 1$ to time $t = 2$.
   c. (12 pts) Find the curvature of the particle’s path at the point $(2, 1, 0)$.
   d. (10 pts) Find the normal and tangential components of the particle’s acceleration at the point $(2, 1, 0)$. Label your answers.
   e. (9 pts) Find $\mathbf{T}$, $\mathbf{N}$, $\mathbf{B}$ for the particle’s path at the point $(2, 1, 0)$.
   f. (5 pts) Find the osculating plane for the particle’s path at the point $(2, 1, 0)$.
7 (6 pts). Find the graph of given vector-valued function.

a. \( \langle 2 - 3t, 2t + 1, 4t \rangle \)  
b. \( \langle \sin(10t), \cos(10t), t \rangle \)  
c. \( \langle 2 - t^2, t, t^2 - 1 \rangle \)  
d. \( \langle \sin t, \cos t, \frac{1}{2} \sin t - \frac{3}{4} \cos t \rangle \)  
e. \( \langle \sqrt{t} \sin t, \sqrt{t} \cos t, \sqrt{t} \rangle \)  
f. \( \langle \sin t, \cos t, \sin(4t) \rangle \)
1. (Source: 12.1.18) Complete the square:

\[ x^2 - 2x + y^2 + z^2 + 4z = 4 \]
\[ x^2 - 2x + 1 + y^2 + z^2 + 4z + 4 = 4 + 1 + 4 \]
\[ (x - 1)^2 + y^2 + (z + 2)^2 = 9 \]

Center is \((1, 0, -2)\) and radius is 3.

2a. (6 pts) \(4x^2 = z\) is a cylinder with a parabolic cross section at every \(y\)-value. See below, left.

2b. (6 pts) \(4x^2 + y^2 = z\) is an elliptical parabola. Its cross-sections at \(x\)- or \(y\)-values are parabolas, and at \(z\)-values are ellipses. See above, right.

3a. (5 pts) \(\cos \theta = \frac{u \cdot v}{|u||v|} = \frac{-2 + 6 - 2}{\sqrt{9\sqrt{14}}} = \frac{2}{3\sqrt{14}}\).

3b. (4 pts) \(\text{proj}_u v = \frac{u \cdot v}{u \cdot u} u = \frac{2}{9} \langle -2, 2, 1 \rangle\).

3c. (5 pts) \(u \times v =\)

\[
\begin{vmatrix}
  \mathbf{i} & \mathbf{j} & \mathbf{k} \\
  -2 & 2 & 1 \\
  1 & 3 & -2 \\
\end{vmatrix}
= \mathbf{i} \begin{vmatrix} 2 & 1 \\ 3 & -2 \end{vmatrix} - \mathbf{j} \begin{vmatrix} -2 & 1 \\ 1 & -2 \end{vmatrix} + \mathbf{k} \begin{vmatrix} -2 & 2 \\ 1 & 3 \end{vmatrix} = \langle -7, -3, -8 \rangle
\]

4. (7 pts) The plane is parallel the vectors \(\langle -2, 3, 1 \rangle - \langle 0, 1, 0 \rangle = \langle -2, 2, 1 \rangle\) and \(\langle 1, 4, -2 \rangle - \langle 0, 1, 0 \rangle = \langle 1, 3, -2 \rangle\). The cross product of these, found in Problem 3, is normal to the plane, so the plane is given by

\[-7x - 3(y - 1) - 8z = 0,\]

or, \(7x + 3y + 8z = 3\).

5. (5 pts) The line is parallel the vector \(\langle 4, 0, 1 \rangle - \langle 2, 2, 2 \rangle = \langle 2, -2, -1 \rangle\) and is parametrized by by

\[l(t) = \langle 4, 0, 1 \rangle + t\langle 2, -2, -1 \rangle.\]
6a. (7 pts). (Source: 13.4.12) \( \mathbf{v} = \frac{d\mathbf{r}}{dt} = (2, 2t, t^{-1}) \). \( \mathbf{a} = \frac{d^2\mathbf{r}}{dt^2} = (0, 2, -t^{-2}) \).

Speed \( |\mathbf{v}| = \sqrt{4 + 4t^2 + t^{-2}} = \sqrt{(2t + t^{-1})^2} = 2t + t^{-1} \).

6b. (8 pts). (Source: 13.3.2.3) The length of the curve is 

\[
\ell = \int_1^2 \frac{ds}{dt} dt = \int_1^2 (2t + t^{-1}) dt = \left( t^2 + \ln t \right) \bigg|_1^2 = 4 + \ln 2 - 1 - \ln 1 = 3 + \ln 2.
\]

6c. (12 pts). (Source: 13.3.24,25) \( \mathbf{r} = (2, 1, 0) \) at \( t = 1 \), when \( \mathbf{v} = (2, 2, 1) \) and \( \mathbf{a} = (0, 2, -1) \). Calculate their cross product:

\[
\mathbf{v} \times \mathbf{a} = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
2 & 2 & 1 \\
0 & 2 & -1
\end{vmatrix} = \mathbf{i} \begin{vmatrix} 2 & 1 \\ 0 & -1 \end{vmatrix} - \mathbf{j} \begin{vmatrix} 2 & 2 \\ 0 & 2 \end{vmatrix} + \mathbf{k} \begin{vmatrix} 2 & 2 \\ 0 & 2 \end{vmatrix} = (-4, 2, 4)
\]

and then

\[
\kappa = \frac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|^3} = \frac{2|(-2, 1, 2)|}{|(2, 2, 1)|^3} = \frac{2}{9}.
\]

6d. (10 pts). (Source: 13.4.41)

Solution one: \( a_T = \frac{d\mathbf{r}}{ds} = (2t + t^{-1})' = 2 - t^2 \), which equals 1 at \( t = 1 \).

\( a_N = \kappa \left( \frac{ds}{dt} \right)^2 = \frac{2}{3} |(2, 2, 1)|^2 = 2. \)

Solution two: \( a_T = \frac{\mathbf{v} \cdot \mathbf{a}}{|\mathbf{v}|} = \frac{2}{3} = 1 \), and \( a_N = \frac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|} = \frac{|(-4, 2, 4)|}{|(2, 2, 1)|} = \frac{6}{3} = 2. \)

6e. (9 pts). (Source: 13.3.47,48) \( \mathbf{T} = \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{1}{3} (2, 2, 1) \). \( \mathbf{B} = \frac{\mathbf{v} \times \mathbf{a}}{|\mathbf{v} \times \mathbf{a}|} = \frac{1}{6} (-4, 2, 4) = \frac{1}{3} (-2, 1, 2) \).

\[
\mathbf{N} = \mathbf{B} \times \mathbf{T} = \frac{1}{3} (-2, 1, 2) \times \frac{1}{3} (2, 2, 1) = \frac{1}{9} \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
-2 & 1 & 2 \\
2 & 2 & 1
\end{vmatrix} = \frac{1}{9} \left( \mathbf{i} \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} - \mathbf{j} \begin{vmatrix} 2 & 2 \\ 2 & 1 \end{vmatrix} + \mathbf{k} \begin{vmatrix} 2 & 2 \\ 2 & 1 \end{vmatrix} \right) = \frac{1}{9} (-3, 6, -6) = \frac{1}{3} (-1, 2, -2).
\]

If you chose to normalize \( \frac{d\mathbf{T}}{dt} \) to find \( \mathbf{N} \), you’d find

\[
\mathbf{T} = (2t + t^{-1})^{-1} (2, 2t, t^{-1})
\]

\[
\frac{d\mathbf{T}}{dt} = - (2t + t^{-1})^{-2} (2 - t^{-2}) (2, 2t, t^{-1}) + (2t + t^{-1})^{-1} (0, 2, -t^{-2})
\]

which equals \( \frac{1}{3} (-2, 4, -4) \). To find \( \mathbf{N} \), normalize any positive scalar multiple of this, e.g. \( (-1, 2, -2) \).

f. (5 pts). (Source: 13.3.50) The osculating plane is normal to \( \mathbf{v} \times \mathbf{a} = (-4, 2, 4) \), so its equation is \(-4(x - 2) + 2(y - 1) + 4z = 0\, or\, -2x + y + 2z = -3\).

7. (6 pts). (Source: 13.1.21-26) \( a_5, b_6, c_2, d_1, e_7, f_3. \)