MATH 220–02 (Kunkle), Quiz 8 Name: ______________________________

10 pts, 10 minutes Nov 14, 2023

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1. Find the following. You are not required to express y as a function of x in either part.

a (8 pts). The general solution to the differential equation $y\dfrac{dy}{dx} = x(2-y^2)e^x$.

b (2 pts). The particular solution to the differential equation in a that satisfies $y = 2$ when $x = 0$.

1a.(Source: 9.3.8) Separate variables and integrate:

$$\int \frac{y}{2-y^2}\,dy = \int xe^x\,dx$$

On the left, substitute $w = 2 - y^2$, $dw = -2y\,dy$:

$$\int \frac{y}{2-y^2}\,dy = -\frac{1}{2}\int \frac{1}{w}\,dw = -\frac{1}{2}\ln|w| + B = -\frac{1}{2}\ln|2-y^2| + B$$

On the right, use parts:

$$\begin{aligned} u &= x \quad & dv &= e^x\,dx \\ du &= dx \quad & v &= e^x \end{aligned}$$

and the integral becomes

$$\begin{aligned} \int xe^x\,dx &= \int u\,dv = uv - \int v\,du \\ &= xe^x - \int e^x\,dx = xe^x - e^x + C \end{aligned}$$

By setting the two integrals equal and combining the constants, we obtain the general solution

$$-\frac{1}{2}\ln|2-y^2| = xe^x - e^x + C$$

1b.(Source: 9.3.15) At $x = 0$, $y = 2$:

$$-\frac{1}{2}\ln|-2| = -1 + C \quad\Longrightarrow\quad C = 1 - \frac{1}{2}\ln 2,$$

so the particular solution is

$$-\frac{1}{2}\ln|2-y^2| = xe^x - e^x + 1 - \frac{1}{2}\ln 2.$$