MATH 220–02 (Kunkle), Quiz 3
10 pts, 10 minutes

Name: ______________________________
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1 (10 pts). Integrate: $\displaystyle\int \frac{\sqrt{9-x^2}}{x^2}\,dx$

Solution: 1.(Source: 7.3.4) Use trig substitution. Because we want

$$9 - x^2 = 9 - 9\sin^2\theta = 9(1-\sin^2\theta) = 9\cos^2\theta,$$

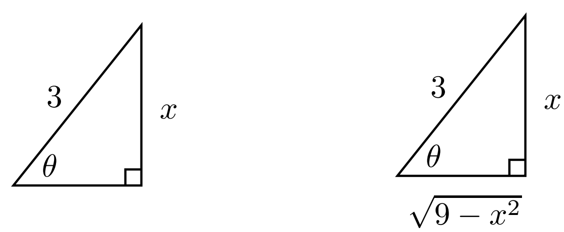
we let

$$x = 3\sin\theta, \quad dx = 3\cos\theta\,d\theta.$$

Now the integral becomes

$$\int \frac{\sqrt{9\cos^2\theta}}{9\sin^2\theta} 3\cos\theta\,d\theta = \int \frac{\cos^2\theta}{\sin^2\theta}\,d\theta = \int \cot^2\theta\,d\theta = \int (\csc^2\theta - 1)\,d\theta = -\cot\theta - \theta + C$$

To rewrite this answer in terms of the original variable x, draw a right triangle with interior angle θ. Label two sides using $\sin\theta = x/3$, and then find the third side by the Pythagorean theorem:



So the integral equals $-\cot\theta - \theta + C = -\dfrac{\sqrt{9-x^2}}{x} - \sin^{-1}(x/3) + C.$