1 (10 pts). Integrate: \( \int \frac{\sqrt{9 - x^2}}{x^2} \, dx \)

\textbf{Solution:} 1.\,(Source: 7.3.4) Use trig substitution. Because we want

\[ 9 - x^2 = 9 - 9 \sin^2 \theta = 9(1 - \sin^2 \theta) = 9 \cos^2 \theta, \]

we let

\[ x = 3 \sin \theta, \quad dx = 3 \cos \theta \, d\theta. \]

Now the integral becomes

\[ \int \frac{\sqrt{9 \cos^2 \theta}}{9 \sin^2 \theta} \, 3 \cos \theta \, d\theta = \int \frac{\cos^2 \theta}{\sin^2 \theta} \, d\theta = \int \cot^2 \theta \, d\theta = \int (\csc^2 \theta - 1) \, d\theta = -\cot \theta - \theta + C \]

To rewrite this answer in terms of the original variable \( x \), draw a right triangle with interior angle \( \theta \). Label two sides using \( \sin \theta = x/3 \), and then find the third side by the Pythagorean theorem:

So the integral equals \(-\cot \theta - \theta + C = -\frac{\sqrt{9 - x^2}}{x} - \sin^{-1}(x/3) + C.\)