MATH 220–02 (Kunkle), Quiz 2 Name: ______________________

10 pts, 10 minutes Sept 5, 2023

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1 (10 pts). Integrate: $\int \cos(\sqrt{x})\,dx$

Solution:

1(10 pts). (Source: 7.1.37) Integrate: $\int \cos(\sqrt{x})\,dx$

First, substitute $t = \sqrt{x}$. There are two different ways to go about this substitution.

Either rewrite the substitution as $x = t^2$. Then $dx = 2t\,dt$, and the integral becomes

$$\int \cos(\sqrt{x})\,dx = \int \cos(t)2t\,dt = \int 2t\cos t\,dt.$$

Or take the differential of $t = \sqrt{x}$ to obtain $dt = \frac{1}{2}\frac{1}{\sqrt{x}}\,dx$. Multiply top and bottom of the integrand by $2\sqrt{x}$:

$$\int \cos(\sqrt{x})\,dx = \int \frac{2\sqrt{x}}{2\sqrt{x}}\cos(\sqrt{x})\,dx = \int 2\sqrt{x}\cos(\sqrt{x})\,\frac{dx}{2\sqrt{x}}$$

and then rewrite the result in terms of t:

$$\int 2t\cos t\,dt$$

Now integrate by parts:

$$\begin{aligned} u &= 2t & dv &= \cos t\,dt \\ du &= 2\,dt & v &= \sin t \end{aligned}$$

$$\int u\,dv = uv - \int v\,du$$

$$\begin{aligned} \int 2t\cos t\,dt &= 2t\sin t - \int 2\sin t\,dt \\ &= 2t\sin t + 2\cos t + C \end{aligned}$$

Rewrite in terms of the original variable x and we're done:

$$\int \cos(\sqrt{x})\,dx = 2\sqrt{x}\sin(\sqrt{x}) + 2\cos(\sqrt{x}) + C$$