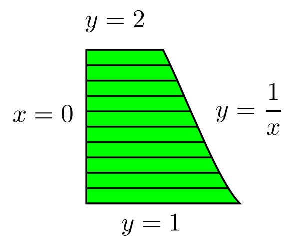
MATH 220–02 (Kunkle), Quiz 1 Name: ____________________
10 pts, 10 minutes Aug 29, 2023

1 (10 pts). Let R denote the region in the first quadrant bounded by the curves $x = 0$, $y = 1$, $y = 2$, and $y = 1/x$.

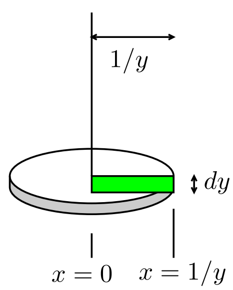
a. Express the volume generated when R is rotated about the line $x = 0$ as a definite integral, but **do not integrate**.

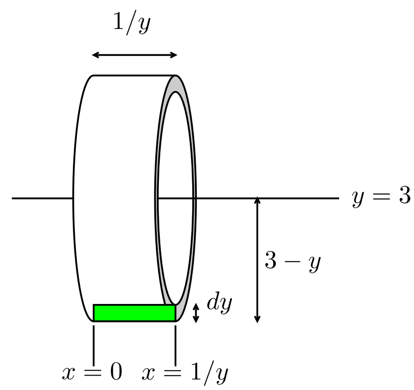
b. Express the volume generated when R is rotated about the line $y = 3$ as a definite integral, but **do not integrate**.

Solution: (Source: Stewart 8e, 6.2.2, 6.3.9) You don't need a highly accurate graph of R to answer this question but you should know that $y = 1$ and $y = 2$ are horizontal lines and that $x = 0$ is the y-axis, and that $y = 1/x$ is a decreasing function of x. R must look something like this (after slicing horizontally):



Rotating each such rectangle about a vertical line (as in part a.) results in a disc. Rotating about a horizontal line (part b.) makes a shell:





So the volumes are:

$$a. \quad V = \int dV = \int_1^2 \pi \frac{1}{y^2}\, dy \qquad b. \quad V = \int dV = \int_1^2 2\pi(3-y)\left(\frac{1}{y}\right) dy$$

(done)

Comment: The solution is more difficult if you slice the region vertically, since the y value at the top of a rectangle is either 2 or $1/x$, depending on whether x is less than or greater than $1/2$. Unsimplifed answers are

$$a. \quad V = \int_0^{1/2} 2\pi x(2-1)\, dx + \int_{1/2}^1 2\pi x\left(x^{-1}-1\right) dx$$

$$b. \quad V = \int_0^{1/2} \pi\big((3-1)^2 - (3-2)^2\big)\, dx + \int_{1/2}^1 \pi\left((3-1)^2 - \left(3-x^{-1}\right)^2\right) dx$$