MATH 203–01&03 (Kunkle), Quiz 6 Name: ____________________
10 pts, 10 minutes Mar 2, 2023

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1 (10 pts). Prove or disprove that the set

$$X = \{\, a(t-1)^2 \mid a \in \mathbb{R} \,\}$$

is a subspace of $\mathbb{P}$, the vector space of all polynomials $p(t)$.

The shortest solution is this:

Solution: $X = \operatorname{span}\left\{(t-1)^2\right\}$, so it is a vector space by Theorem 1, Section 4.1. **(done)**

You could also prove X is a subspace using only the definition of subspace:

Solution:

$\mathbf{0}$ is in X, since $\mathbf{0} = 0(t-1)^2$.

To show that X is closed under addition, note that the sum of any $a(t-1)^2$ and $b(t-1)^2$ in X is also in X, since

$$a(t-1)^2 + b(t-1)^2 = (a+b)(t-1)^2.$$

X is closed under scalar multiplication since, if $a(t-1)^2$ is in X and c is a scalar, then

$$c\big(a(t-1)^2\big) = (ca)(t-1)^2$$

is also in X.

Since $\mathbf{0} \in X$ and X is closed under vector addition and scalar multiplication, X is a subspace of $\mathbb{P}$. **(done)**

By the way, using some facts from calculus II, one can show that X is the kernel of the linear transformation

$$T : \mathbb{P}_2 \to \mathbb{R}^2 : p(t) \mapsto \begin{bmatrix} p(1) \\ p'(1) \end{bmatrix}$$

and therefore X must be a subspace. See Section 4.2 of the review notes at https://kunklet.people.cofc.edu/MATH203/203review.pdf.