MATH 203–03 (Kunkle), Quiz 2 Name: ______________________

10 pts, Take-home Due 9:00pm, Feb 3, 2023

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1 (10 pts).

a. Determine whether the vector $\begin{bmatrix} 1 \\ 12 \\ 25 \end{bmatrix}$ is in the span of the vectors

$$V = \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ -8 \\ -20 \end{bmatrix} \right\}.$$

b. Based on your work, would you say that the vectors V span $\mathbb{R}^3$? Why or why not?

Solution:

1a.(Source: 1.3.11-12) The question asks whether the vector equation

$$x_1 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ -8 \\ -20 \end{bmatrix} = \begin{bmatrix} 1 \\ 12 \\ 25 \end{bmatrix} \tag{1}$$

is consistent. Augment and perform the forward phase of row reduction:

aug.'d matrix	**row op.**	**result**	**row op.**	**result**
$\begin{matrix} 1 & 1 & 0 & 1 \\ 2 & 3 & -8 & 12 \\ 1 & 4 & -20 & 25 \end{matrix}$	$\mathbf{r}_2 \leftarrow \mathbf{r}_2 - 2\,\mathbf{r}_1$ $\mathbf{r}_3 \leftarrow \mathbf{r}_3 - \mathbf{r}_1$	$\begin{matrix} 1 & 1 & 0 & 1 \\ 0 & 1 & -8 & 10 \\ 0 & 3 & -20 & 24 \end{matrix}$	$\mathbf{r}_3 \leftarrow \mathbf{r}_3 - 3\,\mathbf{r}_1$	$\begin{matrix} 1 & 1 & 0 & 1 \\ 0 & 1 & -8 & 10 \\ 0 & 0 & 4 & -6 \end{matrix}$

End forward phase. The reduced augmented matrix, now in row echelon form, doesn't have a pivot in the last column, so the system (1) is consistent.

1b.(Source: 1.4.19-20)

$$\text{ref} \begin{bmatrix} 1 & 1 & 0 \\ 2 & 3 & -8 \\ 1 & 4 & -20 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -8 \\ 0 & 0 & 4 \end{bmatrix}$$

has a pivot in every row, so V spans $\mathbb{R}^3$.

Comment:

If you're asked a question, make sure you answer the question. Don't just stop when you've decided what that answer is. A good solution to part a. should clearly state that the vector *is* in the span of V (as well as the work that leads to that conclusion).

b. If you are asked to explain your answer to a question, be careful that you don't simply explain what the question means. For example,

Question: "Does the equation $x^2 = 1$ have more than one solution? Explain."

Insufficient answer: "Yes, because there's more than one x that satisfies the equation."

Sufficient answer: "Yes. Both $x = 1$ and $x = -1$ satisfy the equation."

MATH 203–01 (Kunkle), Quiz 2 | Name: ______________________
10 pts, Take-home | Due 9:00pm, Feb 3, 2023

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1 (10 pts).

a. Determine whether the vector $\begin{bmatrix}-1\\0\\5\end{bmatrix}$ is in the span of the vectors

$$V = \left\{ \begin{bmatrix}1\\2\\1\end{bmatrix}, \begin{bmatrix}1\\3\\4\end{bmatrix}, \begin{bmatrix}5\\15\\20\end{bmatrix} \right\}.$$

b. Based on your work, would you say that the vectors V span $\mathbb{R}^3$? Why or why not?

Solution:

1a.(Source: 1.3.11-12) The question asks whether the vector equation

$$x_1 \begin{bmatrix}1\\2\\1\end{bmatrix} + x_2 \begin{bmatrix}1\\3\\4\end{bmatrix} + x_3 \begin{bmatrix}5\\15\\20\end{bmatrix} = \begin{bmatrix}-1\\0\\5\end{bmatrix} \tag{0}$$

is consistent. Augment and perform the forward phase of row reduction:

aug.'d matrix	row op.	result	row op.	result
$\begin{matrix}1 & 1 & 5 & -1\\ 2 & 3 & 15 & 0\\ 1 & 4 & 20 & 5\end{matrix}$	$\mathbf{r}_2 \leftarrow \mathbf{r}_2 - 2\,\mathbf{r}_1$ $\mathbf{r}_3 \leftarrow \mathbf{r}_3 - \mathbf{r}_1$	$\begin{matrix}1 & 1 & 5 & -1\\ 0 & 1 & 5 & 2\\ 0 & 3 & 15 & 6\end{matrix}$	$\mathbf{r}_3 \leftarrow \mathbf{r}_3 - 3\,\mathbf{r}_1$	$\begin{matrix}1 & 1 & 5 & -1\\ 0 & 1 & 5 & 2\\ 0 & 0 & 0 & 0\end{matrix}$

End forward phase. The reduced augmented matrix, now in row echelon form, doesn't have a pivot in the last column, so the system (0) is consistent.

1b.(Source: 1.4.19-20)

$$\operatorname{ref} \begin{bmatrix}1 & 1 & 5\\ 2 & 3 & 15\\ 1 & 4 & 20\end{bmatrix} = \begin{bmatrix}1 & 1 & 5\\ 0 & 1 & 5\\ 0 & 0 & 0\end{bmatrix}$$

does not have a pivot in every row, so V does not span $\mathbb{R}^3$. **(done)**

Comment:

If you're asked a question, make sure you answer the question. Don't just stop when you've decided what that answer is. A good solution to part a. should clearly state that the vector *is* in the span of V(as well as the work that leads to that conclusion).

b. If you are asked to explain your answer to a question, be careful that you don't simply explain what the question means. For example,

Question: "Does the equation $x^2 = 1$ have more than one solution? Explain."

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