1 (10 pts).

a. Determine whether the vector \[
\begin{bmatrix}
1 \\
12 \\
25
\end{bmatrix}
\] is in the span of the vectors \[V = \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ -8 \\ -20 \end{bmatrix} \right\}.\]

b. Based on your work, would you say that the vectors \(V\) span \(\mathbb{R}^3\)? Why or why not?

Solution:

1a. (Source: 1.3.11-12) The question asks whether the vector equation
\[
x_1 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ -8 \\ -20 \end{bmatrix} = \begin{bmatrix} 1 \\ 12 \\ 25 \end{bmatrix}
\]
is consistent. Augment and perform the forward phase of row reduction:

<table>
<thead>
<tr>
<th>aug.'d matrix</th>
<th>row op.</th>
<th>result</th>
<th>row op.</th>
<th>result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 1 0 1</td>
<td>(r_2 \leftarrow r_2 - 2r_1)</td>
<td>1 1 0 1</td>
<td>(r_3 \leftarrow r_3 - 3r_1)</td>
<td>1 1 0 1</td>
</tr>
<tr>
<td>2 3 -8 12</td>
<td>(r_3 \leftarrow r_3 - r_1)</td>
<td>0 1 -8 10</td>
<td>0 1 -8 10</td>
<td></td>
</tr>
<tr>
<td>1 4 -20 25</td>
<td></td>
<td>0 3 -20 24</td>
<td>0 0 4 -6</td>
<td></td>
</tr>
</tbody>
</table>

End forward phase. The reduced augmented matrix, now in row echelon form, doesn’t have a pivot in the last column, so the system (1) is consistent.

1b. (Source: 1.4.19-20)

\[
\text{ref } \begin{bmatrix} 1 & 1 & 0 \\ 2 & 3 & -8 \\ 1 & 4 & -20 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -8 \\ 0 & 0 & 4 \end{bmatrix}
\]

has a pivot in every row, so \(V\) spans \(\mathbb{R}^3\).

Comment:

If you’re asked a question, make sure you answer the question. Don’t just stop when you’ve decided what that answer is. A good solution to part a should clearly state that the vector \(\) is in the span of \(V\) (as well as the work that leads to that conclusion).

b. If you are asked to explain your answer to a question, be careful that you don’t simply explain what the question means. For example,

Question: “Does the equation \(x^2 = 1\) have more than one solution? Explain.”

Insufficient answer: “Yes, because there’s more than one \(x\) that satisfies the equation.”

Sufficient answer: “Yes. Both \(x = 1\) and \(x = -1\) satisfy the equation.”
1 (10 pts).

a. Determine whether the vector \[ \begin{bmatrix} -1 \\ 0 \\ 5 \end{bmatrix} \] is in the span of the vectors

\[ V = \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 5 \\ 15 \\ 20 \end{bmatrix} \right\}. \]

b. Based on your work, would you say that the vectors \( V \) span \( \mathbb{R}^3 \)? Why or why not?

Solution:

1a. (Source: 1.3.11-12) The question asks whether the vector equation

\[ x_1 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} + x_3 \begin{bmatrix} 5 \\ 15 \\ 20 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 5 \end{bmatrix} \] (0)

is consistent. Augment and perform the forward phase of row reduction:

<table>
<thead>
<tr>
<th>aug.'d matrix</th>
<th>row op.</th>
<th>result</th>
<th>row op.</th>
<th>result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 1 5 -1</td>
<td>( r_2 \leftarrow r_2 - 2r_1 )</td>
<td>1 1 5 -1</td>
<td>( r_3 \leftarrow r_3 - 3r_1 )</td>
<td>1 1 5 2</td>
</tr>
<tr>
<td>2 3 15 0</td>
<td>( r_3 \leftarrow r_3 - r_1 )</td>
<td>0 1 5 2</td>
<td></td>
<td>0 0 0 0</td>
</tr>
<tr>
<td>1 4 20 5</td>
<td></td>
<td>0 3 15 6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

End forward phase. The reduced augmented matrix, now in row echelon form, doesn’t have a pivot in the last column, so the system (0) is consistent.

1b. (Source: 1.4.19-20)

\[ \text{ref} \begin{bmatrix} 1 & 1 & 5 \\ 2 & 3 & 15 \\ 1 & 4 & 20 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 5 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{bmatrix} \]

does not have a pivot in every row, so \( V \) does not span \( \mathbb{R}^3 \). (done)

Comment:

If you’re asked a question, make sure you answer the question. Don’t just stop when you’ve decided what that answer is. A good solution to part a. should clearly state that the vector is in the span of \( V \) (as well as the work that leads to that conclusion).

b. If you are asked to explain your answer to a question, be careful that you don’t simply explain what the question means. For example,

Question: “Does the equation \( x^2 = 1 \) have more than one solution? Explain.”

Insufficient answer: “Yes, because there’s more than one \( x \) that satisfies the equation.”

Sufficient answer: “Yes. Both \( x = 1 \) and \( x = -1 \) satisfy the equation.”