MATH 120–11 (Kunkle), Quiz 5 Name: ____________________
10 pts, 10 minutes Feb 29, 2024

1a (8 pts). Find the linearization of $f(x) = \dfrac{2x-1}{x+1}$ at (the x-value) $a = 1$.

1b (2 pts). Use your answer to 1a to approximation $f(1.01)$.
You can leave unfinished arithmetic in your answers.

Solution:

1a.(Source: 3.10.1,122) Find $\dfrac{dy}{dx}$ using either use the quotient rule:

$$\frac{(2x-1)'(x+1)-(x+1)'(2x-1)}{(x+1)^2} = \frac{2(x+1)-(2x-1)}{(x+1)^2} = \frac{3}{(x+1)^2},$$

or rewrite the function as $(2x-1)(x+1)^{-1}$ and use the product and chain rules:

$$(2x-1)'(x+1)^{-1} + (2x-1)(-1)(x+1)^{-2}(x+1)' = 2(x+1)^{-1} - (2x-1)(x+1)^{-2},$$

or $(x+1)^{-2}\big(2(x+1)-(2x-1)\big) = 3(x+1)^{-2}$.
Recall that the linearization of a function $f(x)$ is

$$L(x) = f(a) + f'(a)(x-a).$$

$L(x)$ is the function whose graph is the line tangent to the graph of $f(x)$ at $x = a$.
For our function, at $a = 1$, we have

$$\begin{aligned} L(x) &= f(1) + f'(1)(x-1) \\ &= \frac{2-1}{1+1} + \frac{3}{(1+1)^2}(x-1) \\ &= \frac{1}{2} + \frac{3}{4}(x-1). \end{aligned}$$

See the graphs of $f(x)$ and $L(x)$ at https://www.desmos.com/calculator/wjsiqnyoqz.
1b.

$$f(1.01) \approx L(1.01) = \frac{1}{2} + \frac{3}{4}(1.01-1).$$

You're not required to write this in decimal form, but when we do, we see $L(1.01) = 0.5 + (.75)(.01) = 0.5075$ is very close to $f(1.01) = \frac{1.02}{2.01} = 0.50746287\ldots$.