No notes, books, electronic devices, or outside materials of any kind.

Read each problem carefully and simplify your answers.

Unless otherwise indicated, supporting work will be required on every problem worth more than 3 points.

You are expected to know the values of all trig functions at multiples of $\pi/4$ and of $\pi/6$.

You will not receive credit for using l'Hospital's Rule (a technique learned later in a calculus course) on any problem on this exam.

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1. If a ball is thrown into the air with a velocity of 40 ft/sec, its height t seconds later is given by $s(t) = 40t - 16t^2$.

a(14 pts). Use the definition of the derivative to find the ball's velocity function $s'(a)$. You can use shortcut methods from Chapter 3 to check your work, but to receive credit on this problem, you must calculate the derivative from its definition as a limit.

b(3 pts). What is the ball's instantaneous velocity at time $t = 1$?

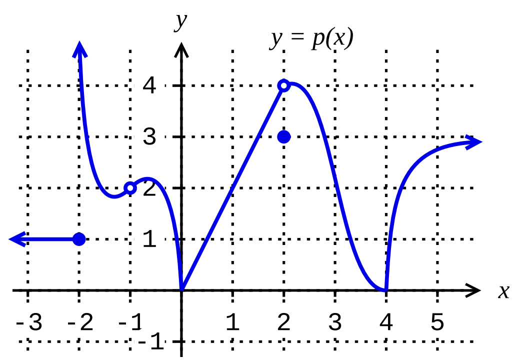
2. Total rainfall is measured in centimeters (cm). During a storm yesterday, I recorded the total rainfall in my yard every ten minutes. Here are some of my measurements,

time	1:40	1:50	2:00	2:10	2:20
total rainfall (cm)	0.0	0.6	2.0	3.2	3.6

a(5 pts). Over what ten-minute interval was the average rate of rainfall (in cm per minute) greatest? What was that average rate over those ten minutes?

b(4 pts). What was the approximate rate of rainfall (cm/min) at time 2:00?

3(13 pts). Use the graph of $p(x)$ to find the following. Supporting work is not required. Your answer to a-h should be ∞, $-\infty$, a number, or "dne."



a. $\lim_{x\to -2} p(x)$ b. $p(-2)$

c. $\lim_{x\to -2^+} p(x)$ d. $\lim_{x\to -1} p(x)$

e. $\lim_{x\to 4} p(x)$ f. $\lim_{x\to \infty} p(x)$

g. $p(-1)$ h. $p(2)$

i. At what x-values, if any, is $p(x)$ discontinuous? At these points, is $p(x)$ continuous from the right, from the left, or neither? Supporting work not required.

4(36 pts). Evaluate the limit or briefly explain why it does not exist.

a. $\displaystyle\lim_{x\to -\infty} \frac{2x^2 - 7x}{x+5}$ b. $\displaystyle\lim_{x\to \infty} \frac{2x+4}{x+3}$ c. $\displaystyle\lim_{x\to \infty} \big(\ln(2x+4) - \ln(x+3)\big)$

d. $\displaystyle\lim_{x\to 1} \frac{\sqrt{3-x} - \sqrt{3x-1}}{x-1}$ e. $\displaystyle\lim_{x\to 1} \frac{x^2+4x-5}{2x^2+3x-5}$ f. $\displaystyle\lim_{x\to 1} \frac{2x-3}{(x-1)^2}$

5a(10 pts). State the precise, ε-δ definition of what it means for $\lim_{x\to r} s(x) = P$.

5b(15 pts). Write an ε-δ proof that $\lim_{x\to -1} (3x+8) = 5$.

1a(14 pts).(Source: 2.7.8,33, 2.7.more.1k) $s'(a) = \lim_{h\to 0} \frac{s(a+h)-s(a)}{h}$

$$= \lim_{h\to 0} \frac{40(a+h)-16(a+h)^2-(40a-16a^2)}{h}$$
$$= \lim_{h\to 0} \frac{40(a+h)-16(a^2+2ah+h^2)-(40a-16a^2)}{h}$$
$$= \lim_{h\to 0} \frac{40a+40h-16a^2-32ah-16h^2-40a+16a^2}{h} = \lim_{h\to 0} \frac{40h-32ah-16h^2}{h}$$
$$= \lim_{h\to 0} \frac{h(40-32a-16h)}{h} = \lim_{h\to 0}(40-32a-16h) = 40-32a.$$

1b(3 pts).(Source: 2.7.8) The object's velocity is the the derivative of its position: $s'(1) = 40-32 = 8$ (feet per second).

2a(5 pts).(Source: 2.1.1) Here are the amounts of rain that fell in each ten-minute period and the average rate of rainfall in each:

time interval	1:40-1:50	1:50-2:00	2:00-2:10	2:10-2:20
rainfall during that interval (cm)	0.6	1.4	1.2	0.4
average rate of rainfall (cm/min)	0.06	0.14	0.12	0.04

Not all these calculations are necessary to answer the question correctly. The period with heaviest rainfall was 1:50-2:00, when the average rate of rainfall was $\frac{1.4}{10} = 0.14$ cm/min. Corrected, Apr 25. Thanks to R.S. for pointing out my error.

2b(4 pts). Average rate of rainfall from 1:50-2:00 was 0.14 cm/min, the average rate of rainfall from 2:00-2:10 was 0.12 cm/min, and their average is 0.13 cm/min. Any one of these is a reasonable approximation to the rate of rainfall at 2:00.

3(13 pts).(Source: 2.2.4-9, 2.6.3-4) a DNE. b1. c1. d2. e0. f3. g DNE. h3.
i. $p(x)$ is discontinuous where it does not exist ($x=-1$), where its limit does not exist ($x=-2$) and where the limit and function value disagree ($x=2$). Since $p(-2) = \lim_{x\to -2^-} p(x)$, p is continuous from the left at -2. It is neither left- nor right-continuous at -1 or at 2.

4a(4 pts).(Source: 2.6.42) The limit at $-\infty$ of a rational function is determined by its lead terms in top and bottom: $\lim_{x\to -\infty} \frac{2x^2-7x}{x+5} = \lim_{x\to -\infty} \frac{2x^2}{x} = \lim_{x\to -\infty} 2x = -\infty$.

4b(4 pts).(Source: 2.6.15) By the same method, $\lim_{x\to\infty} \frac{2x+4}{x+3} = \lim_{x\to\infty} \frac{2x}{x} = \lim_{x\to\infty} 2 = 2$.

4c(6 pts).(Source: 2.6.42) Using property 5 of the natural log, p. 17 of the review notes https://kunklet.people.cofc.edu/MATH120/120review.pdf,
$\lim_{x\to\infty} \big(\ln(2x+4) - \ln(x+3)\big) = \lim_{x\to\infty} \ln\left(\frac{2x+4}{x+3}\right) = \ln\left(\lim_{x\to\infty}\left(\frac{2x+4}{x+3}\right)\right) = \ln 2.$

4d(9 pts).(Source: 2.3.25) Rationalize the numerator by means of the conjugate:

$$\lim_{x\to 1}\left(\frac{\sqrt{3-x}-\sqrt{3x-1}}{x-1}\right)\left(\frac{\sqrt{3-x}+\sqrt{3x-1}}{\sqrt{3-x}+\sqrt{3x-1}}\right)$$
$$=\lim_{x\to 1}\frac{(3-x)-(3x-1)}{(x-1)(\sqrt{3-x}+\sqrt{3x-1})}=\lim_{x\to 1}\frac{4-4x}{(x-1)(\sqrt{3-x}+\sqrt{3x-1})}$$
$$=\lim_{x\to 1}\frac{-4(x-1)}{(x-1)(\sqrt{3-x}+\sqrt{3x-1})}=\lim_{x\to 1}\frac{-4}{\sqrt{3-x}+\sqrt{3x-1}}=\frac{-4}{2\sqrt{2}}=-\sqrt{2}$$

4e(7 pts).(Source: 2.3.15,16) Find and cancel the common factor.

$$\lim_{x\to 1}\frac{x^2+4x-5}{2x^2+3x-5}=\lim_{x\to 1}\frac{(x-1)(x+5)}{(x-1)(2x+5)}=\lim_{x\to 1}\frac{x+5}{2x+5}=\frac{6}{7}.$$

4f(6 pts).(Source: 2.2.33) As $x\to 1$, $\frac{2x-3}{(x-1)^2}$ goes to "$\frac{-1}{0}$," indicating that the limit is infinite. To decide between $\pm\infty$, observe that the numerator must be negative (at least for x close to 1), because it approaches -1. The denominator is a square must be positive (except when $x=1$). Therefore the fraction is negative, and its limit must be $-\infty$.

5a(10 pts).(Source: Students were told in class to be prepared to state this definition from section 2.4.) $\lim\limits_{x\to r} s(x)=P$ means that for every posiive number ε, there's a corresponding positive number δ for which $|s(x)-P|<\varepsilon$ whenever $0<|x-r|<\delta$.

In this definition and the proof below, **wording matters**. Small changes in wording can significantly change the meaning of what you've written.

5b(15 pts).(Source: 2.4.21,22, 2.4.re7, 2.4.more.1g) Here's the thinking I did before writing my proof: $|3x+8-5|=|3x-3|=|3||x-1|=3|x-1|$. We can make this less than ε by taking $|x-1|<\frac{1}{3}\varepsilon$.
Proof: Suppose that $\varepsilon>0$, and choose $\delta=\frac{1}{3}\varepsilon$. Then

$$|3x+8-5|=|3x-3|=|3||x-1|=3|x-1|<3\delta=\varepsilon$$

whenever $0<|x-1|<\delta$. QED