1 (20 pts). Find the values of the given trig functions at the given values of $t$. Write “DNE” when appropriate. Supporting work not required on this problem.

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>$\frac{\pi}{3}$</th>
<th>$\frac{5\pi}{6}$</th>
<th>$-\frac{17\pi}{6}$</th>
<th>$\frac{3\pi}{4}$</th>
<th>$-\frac{17\pi}{2}$</th>
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</thead>
<tbody>
<tr>
<td>$\sin t$</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$\cos t$</td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tan t$</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\cot t$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sec t$</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$\csc t$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

2 (3 pts). Convert $-64^\circ$ to radians.

3 (3 pts). Find the arclength subtended by a central angle of 2 radians in a circle of radius 3 cm.

4 (26 pts). Find the following.
   a. $\cos \left(\frac{3\pi}{8}\right)$
   b. $\sin \left(-\frac{5\pi}{12}\right)$
   c. $\tan^{-1}(\sqrt{3})$
   d. $\sin^{-1}\left(-\frac{1}{2}\right)$
   e. $\sin \left(\tan^{-1}(2)\right)$
   f. $\cos^{-1}\left(\cos\left(\frac{7\pi}{8}\right)\right)$

5 (13 pts). Sketch one cycle of the graph of $y = 2 - \cos\left(3x - \frac{\pi}{4}\right)$. Draw the axes where you wish and label hashmarks so as to clearly indicate every point in your cycle where the cosine equals 0, 1, or −1. What is the amplitude of this function?

6 (9 pts). Sketch one cycle of the graph of $y = 2\sec\left(3x - \frac{\pi}{4}\right)$. Draw the axes where you wish and label hashmarks so as to clearly indicate every point in your cycle where the secant equals 1 or −1. Give the equation(s) of any asymptotes that occur in your cycle. What is the period of this function?

7 (8 pts). Suppose that $\sin x = -\frac{1}{3}$ and that $\frac{\pi}{2} < x < \frac{3\pi}{2}$. Find the values of the other five trig functions at $x$.

8 (5 pts). Use the substitution $x = b\tan \theta$, $-\pi/2 < \theta < \pi/2$ to rewrite the algebraic expression $\sqrt{b^2 + x^2}$ as a trigonometric expression without radicals. You may not assume that $b > 0$.

9 (13 pts). Find all real solutions $x$ to the given equation.
   a. $\tan(3x) = \sqrt{3}$
   b. $\cos^2 x + 3 \cos x = 1 + \sin^2 x$
1. (Source: 4.2.15-32, 4.2.more.1, 4.4.3-18) Determine the reference number of each angle and the quadrant containing its terminal point. Draw a vertical line from the terminal point to the x-axis and a line from the terminal point to the origin. Look for these two triangles. You were not required to rationalize denominators.

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>$\frac{\pi}{3}$</th>
<th>$\frac{5\pi}{6}$</th>
<th>$-\frac{17\pi}{6}$</th>
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<tr>
<td>quadrant</td>
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<td>I</td>
<td>II</td>
<td>III</td>
<td>II</td>
<td>South Pole</td>
</tr>
<tr>
<td>ref. number</td>
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<td>$\frac{\pi}{3}$</td>
<td>$\frac{\pi}{6}$</td>
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<td>$-1/2$</td>
<td>$1/\sqrt{2}$</td>
<td>$-1$</td>
</tr>
<tr>
<td>cos $t$</td>
<td>1</td>
<td>$1/2$</td>
<td>$-\sqrt{3}/2$</td>
<td>$-\sqrt{3}/2$</td>
<td>$-1/\sqrt{2}$</td>
<td>0</td>
</tr>
<tr>
<td>tan $t = \frac{\sin t}{\cos t}$</td>
<td>DNE</td>
<td>$\sqrt{3}$</td>
<td>$-1/\sqrt{3}$</td>
<td>$1/\sqrt{3}$</td>
<td>$-1$</td>
<td>DNE</td>
</tr>
<tr>
<td>cot $t = \frac{\cos t}{\sin t}$</td>
<td>DNE</td>
<td>$1/\sqrt{3}$</td>
<td>$-\sqrt{3}$</td>
<td>$\sqrt{3}$</td>
<td>$-1$</td>
<td>0</td>
</tr>
<tr>
<td>sec $t = \frac{1}{\cos t}$</td>
<td>1</td>
<td>2</td>
<td>$-2/\sqrt{3}$</td>
<td>$-2/\sqrt{3}$</td>
<td>$-\sqrt{2}$</td>
<td>DNE</td>
</tr>
<tr>
<td>csc $t = \frac{1}{\sin t}$</td>
<td>DNE</td>
<td>$2/\sqrt{3}$</td>
<td>2</td>
<td>$-2$</td>
<td>$\sqrt{2}$</td>
<td>$-1$</td>
</tr>
</tbody>
</table>

2. (Source: 4.1.28, 31) $-64^\circ \cdot \frac{\pi}{180^\circ} = -\frac{16\pi}{45}$ radians.

3. (Source: 4.1.69, 70) Arclength = $s = \theta \cdot r = 2 \cdot 3 = 6$ cm.

4a. (Source: 4.5.37) Use the half-angle identity $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$ with $x = \frac{3\pi}{8}$ and get

$$
\cos^2 \frac{3\pi}{8} = \frac{1}{2}\left(1 + \cos \left(\frac{3\pi}{4}\right)\right) = \frac{1}{2}\left(1 + \frac{-1}{\sqrt{2}}\right).
$$

(We found $\cos \left(\frac{3\pi}{4}\right)$ in the table above.) Since $\frac{3\pi}{8}$ is in Quadrant I, we expect its cosine to be positive. Therefore take the positive root to find $\cos \frac{3\pi}{8} = \sqrt{\frac{1}{2} \left(1 - \frac{1}{\sqrt{2}}\right)}$.

4b. (Source: 4.5.8) $\sin \left(-\frac{5\pi}{12}\right) = \sin \left(\frac{5\pi}{12} - \frac{9\pi}{12}\right) = \sin \left(\frac{5\pi}{12}\right) \cos \left(\frac{9\pi}{12}\right) - \cos \left(\frac{5\pi}{12}\right) \sin \left(\frac{9\pi}{12}\right)$

(See table above for these sines and cosines.)

$$
\frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} - \frac{1}{2} \cdot \frac{1}{\sqrt{2}} = \frac{-\sqrt{3} - 1}{2\sqrt{2}}.
$$

4c. (Source: 4.7.2) $\tan^{-1}(\sqrt{3})$ is the unique angle in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ whose tangent is $\sqrt{3}$. (We saw this angle in the table above.) $\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$.

4d. (Source: 4.7.more.2g) $\sin^{-1} \left(-\frac{1}{2}\right)$ is the unique angle in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ whose sine is $-\frac{1}{2}$, so $\sin^{-1} \left(-\frac{1}{2}\right) = -\frac{\pi}{6}$. 

4e. (Source: 4.7.more.7m) \( \theta = \tan^{-1}(2) \) is the angle in \((-\frac{\pi}{2}, \frac{\pi}{2})\) whose tangent is 2. Since its tangent is positive, we know that \( \theta \) is in Quadrant I. Use \( \tan \theta = 2 \) to label the horizontal and vertical legs of the triangle in Quadrant I. (See the figure on the right below.) Find the missing side (\( = \sqrt{5} \)) by the Pythagorean theorem. Then sine of \( \theta \) is \( y/r = 2/\sqrt{5} \).

You can also solve this problem from the Pythagorean identity \( \sec^2 \theta = \tan^2 \theta + 1 = 2^2 + 1 = 5 \). Since \( \theta \) is in Quadrant I, its secant is positive, and so \( \sec \theta = \sqrt{5} \). Then \( \sin \theta = \tan \theta \cos \theta = 2 \cdot \frac{1}{\sqrt{5}} \).

4f. (Source: 4.7.more.5q) \( \frac{7\pi}{5} \) is in Quadrant III and its cosine is the \( x \)-coordinate of its terminal point. \( \cos^{-1}(\cos(\frac{7\pi}{5})) \) is the angle \( \theta \) in \([0, \pi]\) for which \( \cos \theta = \cos(\frac{7\pi}{5}) \). Since the angle between the terminal side of \( \frac{7\pi}{5} \) and the \( x \)-axis is \( \frac{2\pi}{5} \), \( \theta \) must be \( \frac{3\pi}{5} \). (See the figure on the left above.)

5. (Source: 4.3.31-36,43) The function \( y = 2 - \cos(3x - \frac{\pi}{4}) \) will go through one cycle when the angle inside the cosine goes from 0 to \( 2\pi \).

\[
0 \leq 3x - \frac{\pi}{4} \leq 2\pi \\
\frac{\pi}{4} \leq 3x \leq \frac{9\pi}{4} \\
\frac{\pi}{12} \leq x \leq \frac{9\pi}{12}
\]

So, our cycle of the cosine will start at \( x = \frac{\pi}{12} \) and end at \( x = \frac{9\pi}{12} \). The negative sign reflects the graph of cosine across the \( x \)-axis, and the 2 shifts the curve up two units, so that the new centerline of the curve is \( y = 2 \). The minimum value of \( y \) is 1, and the maximum value is 3. The amplitude of the this function is 1.

6. (Source: 4.4.39,41) \( 2 \sec(3x - \frac{\pi}{4}) \) will go through one cycle when \( \frac{\pi}{12} \leq x \leq \frac{9\pi}{12} \), as in Problem 5. The \textbf{period} is the length of this interval, or \( \frac{8\pi}{3} \).

\[
\text{(centerline)}
\]
sec $\theta$ is the reciprocal of $\cos \theta$, so the secant is positive when the cosine is positive, negative when the cosine is positive, $\pm 1$ when the cosine is $\pm 1$, and has a vertical asymptote when the cosine has a zero. The 2 stretches the graph vertically.

7. (Source: 4.2.3, 4.4.23) Since $\frac{\pi}{2} < x < \frac{3\pi}{2}$ and $\sin x < 0$, we know that $x$ is in Quadrant III. Use $\sin x = -1/3$ to label the vertical and hypotenuse of the triangle in Quadrant III. Find the missing side by the Pythagorean theorem.

$$\sin x = -\frac{1}{3}, \quad \tan x = \frac{1}{2\sqrt{2}}, \quad \csc x = -3, \quad \cos x = -\frac{2\sqrt{2}}{3}, \quad \cot x = 2\sqrt{2}, \quad \sec x = -\frac{3}{2\sqrt{2}}$$

8. (Source: 4.4.48) $\sqrt{b^2 + (b \tan \theta)^2} = \sqrt{b^2 + b^2 \tan^2 \theta} = \sqrt{b^2(1 + \tan^2 \theta)} = \sqrt{b^2 \sec^2 \theta} = |b \sec \theta| = |b| \sec \theta$. This last equality is due to the fact that $\sec \theta > 0$, because $-\pi/2 < \theta < \pi/2$. Note that we can’t say that $\sqrt{b^2} = b$ unless we assume that $b \geq 0$.

9a. (Source: 4.8.30) $\tan(3x) = \sqrt{3}$ means that the terminal side of the angle $3x$ has slope $\sqrt{3}$. See figure on left below.

$$3x = \frac{\pi}{3} + 2\pi n \text{ or } \frac{4\pi}{3} + 2\pi n, \text{ and therefore } x = \frac{\pi}{9} + \frac{2\pi n}{3} \text{ or } \frac{4\pi}{9} + \frac{2\pi n}{3}.$$