

# 1 Balanced (Part) Ternary Designs

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## 1.1 Definitions and Example

A *balanced ternary design*  $\text{BTD}(V, B; \rho_1, \rho_2, R; K, \Lambda)$ , is an arrangement of  $V$  elements into  $B$  multisets, or *blocks*, each of cardinality  $K$  ( $K \leq V$ ), satisfying

1. Each element appears  $R = \rho_1 + 2\rho_2$  times altogether, with multiplicity one in exactly  $\rho_1$  blocks, with multiplicity two in exactly  $\rho_2$  blocks.
2. Every pair of distinct elements appears  $\Lambda$  times; i.e., if  $m_{vb}$  is the multiplicity of the  $v$ th element in the  $b$ th block, then for every pair of distinct elements  $v$  and  $w$ , we have  $\sum_{b=1}^B m_{vb}m_{wb} = \Lambda$ .

A *balanced part ternary design*  $\text{BPTD}(V; b_1, b_2, B; \rho_1, \rho_2, R; K, \Lambda)$  is a BTD that also satisfies

3. There exist exactly  $b_2 = B - b_1$  blocks each containing at least one element of multiplicity two.

Hence, every BTD is a BPTD for some choice of  $b_1$  and  $b_2$ .

**1.1 Example** A  $\text{BPTD}(4; 1,8,9; 3,3,9; 4,7)$ .

1	1	1	2	2	3	3	3	1
1	1	1	2	2	3	3	3	2
2	3	4	3	4	1	1	4	3
2	4	4	4	4	2	2	4	4

## 1.2 A BPTD Parameter List

**1.2 Remark** Based on the list in [3], the following is a table of the known and possible values of  $b_2$  for each  $V; B; \rho_1, \rho_2, R; K, \Lambda$ . It includes only parameters for which  $\min(K, \Lambda) > 3$ , since otherwise  $b_2 = V\rho_2$ .

**1.3 Table** Col. 1 shows, in parentheses, the row number of these parameters in [3]. Col. 9 contains the values of  $b_2$  for which a BPTD is known to exist. Col. 10 shows the values of  $b_2$  for which existence is in question. See also [3] and [6, (3.2)] throughout.

# ([3])	$V$	$B$	$\rho_1$	$\rho_2$	$R$	$K$	$\Lambda$	known $b_2$ 's	open $b_2$ 's	reference
1 (7)	5	5	1	2	5	5	4	5	none	[6, (5.3),examples]
2 (11)	8	8	4	1	6	6	4	8	none	[7, (2.1)]
3 (14)	5	6	2	2	6	5	5	5	none	[6, (5.3)]
4 (18)	11	11	5	1	7	7	4	11	none	[7, (2.7)]
5 (22)	5	7	3	2	7	5	6	5	none	[6, (5.3)]
6 (23)	4	7	1	3	7	4	5	6	none	[6, (5.2)]
7 (24)	10	10	1	3	7	7	4	10	none	[7, (2.6)]
8 (25)	7	7	1	3	7	7	6	7	none	[7, (2.6)]

## 2 Balanced (Part) Ternary Designs

# ([3])	$V$	$B$	$\rho_1$	$\rho_2$	$R$	$K$	$\Lambda$	known $b_2$ 's	open $b_2$ 's	reference
9 (30)	10	10	6	1	8	8	6	10	none	[7, (2.1)]
10 (33)	6	12	4	2	8	4	4	8, 10..12	none	[7, (2.4),(2.11)]
11 (34)	5	10	4	2	8	4	5	9, 10	none	[7, (2.4),(2.11)]
12 (35)	5	8	4	2	8	5	7	5	none	[6, (5.3)]
13 (37)	4	8	2	3	8	4	6	6, 8	none	[6, (5.2),examples]
14 (38)	7	8	2	3	8	7	7	7	none	[6, (6.1)]
15 (43)	14	18	7	1	9	7	4	none	14	[7, (2.2)], [4]
16 (45)	15	15	7	1	9	9	5	15	none	[7, (2.7)]
17 (46)	11	11	7	1	9	9	7	11	none	[7, (2.1)]
18 (49)	5	9	5	2	9	5	8	5, 9	none	[6, (5.3),examples]
19 (50)	18	18	5	2	9	9	4	18	none	[7, (2.11)]
20 (55)	4	9	3	3	9	4	7	6, 8, 9	none	[6, (5.2),examples]
21 (56)	7	9	3	3	9	7	8	7	none	[6, (6.1)], [7, (2.11)]
22 (57)	12	12	3	3	9	9	6	12	none	[7, (2.11)], [1, (3.6)]
23 (59)	9	9	1	4	9	9	8	9	none	[7, (2.6)]
24 (61)	8	20	8	1	10	4	4	4,6,7,8	none	
25 (63)	9	15	8	1	10	6	6	9	none	[4]
26 (66)	23	23	8	1	10	10	4	23	none	[7, (2.2)], [1, (3.6)]
27 (67)	12	12	8	1	10	10	8	12	none	[7, (2.1)]
28 (73)	10	20	6	2	10	5	4	10	11..20	
29 (74)	7	14	6	2	10	5	6	14	7..13	
30 (75)	5	10	6	2	10	5	9	5, 9, 10	none	[6, (5.3),examples]
31 (76)	12	15	6	2	10	8	6	none	6..15	
32 (78)	4	10	4	3	10	4	8	6, 8..10	none	[6, (5.2),examples]
33 (79)	12	20	4	3	10	6	4	none	12..20	
34 (80)	7	10	4	3	10	7	9	7	none	[6, (6.1)], [7, (2.11)], [4]
35 (83)	13	13	4	3	10	10	7	13	none	[1, (3.6)]
36 (84)	9	18	2	4	10	5	4	18	none	
37 (85)	5	10	2	4	10	5	8	10	none	
38 (87)	9	10	2	4	10	9	9	9	none	[6, (6.1)]
39 (91)	16	22	9	1	11	8	5	none	8..16	[7, (2.3)]
40 (94)	28	28	9	1	11	11	4	none	28	[7, (2.2)], [1, (3.6)]
41 (95)	19	19	9	1	11	11	6	19	none	[7, (2.7)]
42 (96)	13	13	9	1	11	11	9	13	none	[7, (2.1)]
43 (98)	5	11	7	2	11	5	10	5, 9, 10	none	[6, (5.3)]
44 (99)	20	22	7	2	11	10	5	none	8..22	
45 (102)	4	11	5	3	11	4	9	6, 8..11	none	[6, (5.2),examples]
46 (103)	7	11	5	3	11	7	10	7	10,11	[6, (6.1)], [7, (2.11)], [4]
47 (104)	27	27	5	3	11	11	4	none	27	[7, (2.2)]
48 (105)	14	14	5	3	11	11	8	14	none	[1, (3.6)]
49 (106)	10	22	3	4	11	5	4	none	22	
50 (107)	5	11	3	4	11	5	9	10	none	[6, (3.7)]
51 (108)	9	11	3	4	11	9	10	9	11	[6, (6.1)]
52 (110)	6	11	1	5	11	6	9	10	none	[7, (2.8),(2.10)]
53 (113)	11	11	1	5	11	11	10	11	none	[7, (2.6)]
54 (121)	27	27	10	1	12	12	5	none	27	[1, (3.6)]

# ([3])	V	B	$\rho_1$	$\rho_2$	R	K	$\Lambda$	known $b_2$ 's	open $b_2$ 's	reference
55 (122)	14	14	10	1	12	12	10	14	none	[7, (2.1)]
56 (128)	9	27	8	2	12	4	4	18	9..17	
57 (129)	5	15	8	2	12	4	8	7..10	none	[7, (2.4)]
58 (130)	5	12	8	2	12	5	11	5, 9, 10	none	[6, (5.3)]
59 (132)	15	30	8	2	12	6	4	30	10..29	
60 (133)	9	18	8	2	12	6	7	18	6..17	
61 (134)	8	16	8	2	12	6	8	8, 10, 16	7, 9, 11..15	[4]
62 (135)	24	32	8	2	12	9	4	none	16..32	[7, (2.2)]
63 (136)	33	33	8	2	12	12	4	none	33	[7, (2.2)], [1, (3.6)]
64 (137)	17	17	8	2	12	12	8	none	17	[1, (3.6)]
65 (144)	7	21	6	3	12	4	5	21	11..20	[7, (2.4),(3.2)]
66 (145)	6	18	6	3	12	4	6	12..18	11	[7, (2.4)]
67 (146)	4	12	6	3	12	4	10	6, 8..12	none	[6, (5.2),examples]
68 (147)	10	20	6	3	12	6	6	20	10..19	
69 (148)	7	14	6	3	12	6	9	7	8..14	[6, (1.6)]
70 (149)	7	12	6	3	12	7	11	7	10..12	[6, (6.1)], [7, (2.11)], [4]
71 (150)	14	21	6	3	12	8	6	none	11..21	
72 (152)	19	19	6	3	12	12	7	19	none	[1, (3.6)]
73 (153)	15	15	6	3	12	12	9	15	none	[5], [1, (3.6)]
74 (157)	8	24	4	4	12	4	4	24	16..23	
75 (158)	5	15	4	4	12	4	7	10, 12..15	none	[6, (1.6)], [4]
76 (159)	5	12	4	4	12	5	10	10,12	none	[6, (3.7)]
77 (160)	14	28	4	4	12	6	4	28	19..27	
78 (161)	20	30	4	4	12	8	4	none	20..30	
79 (162)	12	16	4	4	12	9	8	none	12..16	
80 (163)	9	12	4	4	12	9	11	9, 12	11	[7, examples]
81 (164)	32	32	4	4	12	12	4	none	32	[7, (2.2)]
82 (166)	6	12	2	5	12	6	10	10, 12	none	[7, (2.10)]
83 (168)	11	12	2	5	12	11	11	11	none	[6, (6.1)]
84 (171)	18	26	11	1	13	9	6	none	5..18	
85 (172)	33	39	11	1	13	11	4	none	11..33	[7, (2.2)]
86 (175)	23	23	11	1	13	13	7	23	none	[7, (2.7)]
87 (176)	15	15	11	1	13	13	11	15	none	[7, (2.1)]
88 (179)	8	26	9	2	13	4	5	13	8..12, 14..16	
89 (181)	5	13	9	2	13	5	12	5, 9, 10	none	[6, (5.3)]
90 (182)	22	26	9	2	13	11	6	none	9..26	
91 (183)	39	39	9	2	13	13	4	none	39	[1, (3.6)]
92 (184)	20	20	9	2	13	13	8	20	none	[1, (3.6)]
93 (188)	4	13	7	3	13	4	11	6, 8..12	none	[6, (5.2)]
94 (189)	7	13	7	3	13	7	12	7	10..13	[6, (6.1)], [7, (2.11)], [4]
95 (190)	31	31	7	3	13	13	5	none	31	[1, (3.6)]
96 (192)	16	16	7	3	13	13	10	none	16	[1, (3.6)]
97 (194)	5	13	5	4	13	5	11	10,12,13	none	[6, (3.7)]
98 (195)	9	13	5	4	13	9	12	9, 12	11, 13	#1&#18
99 (197)	12	26	3	5	13	6	5	none	20..26	
100 (198)	6	13	3	5	13	6	11	10, 12, 13	none	[7, (2.10)], [4]

#### 4 Balanced (Part) Ternary Designs

# ([3])	V	B	$\rho_1$	$\rho_2$	R	K	$\Lambda$	known $b_2$ 's	open $b_2$ 's	reference
101 (199)	11	13	3	5	13	11	12	11	13	[6, (6.1)]
102 (200)	4	13	1	6	13	4	9	12	none	[7, (2.8)]
103 (203)	19	19	1	6	13	13	8	19	none	[7, (2.6)]
104 (205)	13	13	1	6	13	13	12	13	none	[7, (2.6)]
105 (207)	6	21	12	1	14	4	8	5, 6	none	[7, (2.4),examples]
106 (208)	10	28	12	1	14	5	6	9	5..8, 10	
107 (209)	18	42	12	1	14	6	4	6	7..18	[8]
108 (214)	37	37	12	1	14	14	5	none	37	[1, (3.6)]
109 (215)	31	31	12	1	14	14	6	31	none	[1, (3.6)]
110 (216)	21	21	12	1	14	14	9	none	21	[1, (3.6)]
111 (218)	16	16	12	1	14	14	12	16	none	[7, (2.1)]
112 (222)	5	14	10	2	14	5	13	5, 9, 10	none	[6, (5.3)]
113 (223)	12	28	10	2	14	6	6	none	8..24	
114 (225)	21	42	10	2	14	7	4	42	14..41	
115 (226)	17	34	10	2	14	7	5	34	12..33	
116 (227)	11	22	10	2	14	7	8	22	8..21	
117 (228)	9	18	10	2	14	7	10	18	6..17	
118 (231)	10	35	8	3	14	4	4	25	15..24, 26..30	
119 (232)	4	14	8	3	14	4	12	6, 8..12	none	[6, (5.2)]
120 (233)	9	21	8	3	14	6	8	16	9..15, 17..21	
121 (234)	14	28	8	3	14	7	6	none	14..28	
122 (235)	7	14	8	3	14	7	13	7	10..14	[6, (6.1)], [7, (2.11)], [4]
123 (236)	24	42	8	3	14	8	4	none	18..42	
124 (237)	25	35	8	3	14	10	5	none	15..35	
125 (238)	45	45	8	3	14	14	4	none	45	[1, (3.6)]
126 (239)	23	23	8	3	14	14	8	none	23	[1, (3.6)]
127 (240)	17	17	8	3	14	14	11	none	17	[1, (3.6)]
128 (243)	5	14	6	4	14	5	12	10, 12..14	none	[6, (3.7)]
129 (244)	20	40	6	4	14	7	4	40	27..39	
130 (245)	16	28	6	4	14	8	6	16	17..28	[8]
131 (246)	27	42	6	4	14	9	4	none	27..42	
132 (247)	9	14	6	4	14	9	13	9, 12	11, 13..14	
133 (248)	30	30	6	4	14	14	6	none	30	[1, (3.6)]
134 (249)	6	14	4	5	14	6	12	10, 12, 14	none	[7, (2.8),(2.10)], [4]
135 (250)	12	21	4	5	14	8	8	none	15..21	
136 (251)	18	28	4	5	14	9	6	none	23..28	
137 (252)	30	42	4	5	14	10	4	none	30..42	
138 (253)	11	14	4	5	14	11	13	11	13..14	[6, (6.1)]
139 (255)	6	21	2	6	14	4	6	21	none	[3], [6, (1.7)]
140 (256)	4	14	2	6	14	4	10	12, 14	none	[7, (2.8)]
141 (257)	19	38	2	6	14	7	4	none	38	[4]
142 (258)	13	26	2	6	14	7	6	26	none	
143 (259)	10	20	2	6	14	7	8	20	none	
144 (261)	7	14	2	6	14	7	12	14	none	
145 (262)	20	28	2	6	14	10	6	none	24..28	
146 (264)	13	14	2	6	14	13	13	13	none	[6, (6.1)]

# ([3])	V	B	$\rho_1$	$\rho_2$	R	K	$\Lambda$	known $b_2$ 's	open $b_2$ 's	reference
147 (266)	18	18	2	6	14	14	10	none	18	[1, (3.6)]
148 (273)	20	30	13	1	15	10	7	none	4..20	
149 (275)	53	53	13	1	15	15	4	53	none	[1, (3.6)]
150 (276)	27	27	13	1	15	15	8	27	none	[7, (2.7)]
151 (277)	17	17	13	1	15	15	13	17	none	[7, (2.1)]
152 (281)	15	45	11	2	15	5	4	30	15..29	
153 (282)	9	27	11	2	15	5	7	9	10..18	
154 (283)	8	24	11	2	15	5	8	16	8..15	
155 (284)	5	15	11	2	15	5	14	5, 9, 10	none	[6, (5.3)]
156 (285)	30	50	11	2	15	9	4	none	15..50	
157 (286)	24	30	11	2	15	12	7	none	8..30	
158 (293)	4	15	9	3	15	4	13	6, 8..12	none	[6, (5.2)]
159 (296)	10	30	9	3	15	5	6	none	15..30	
160 (297)	7	21	9	3	15	5	9	21	11..20	
161 (298)	7	15	9	3	15	7	14	7	10..15	[6, (6.1)], [7, (2.11)], [4]
162 (299)	28	30	9	3	15	14	7	none	12..30	
163 (301)	35	35	9	3	15	15	6	none	35	[1, (3.6)]
164 (302)	18	18	9	3	15	15	12	18	none	[5] [1, (3.6)]
165 (305)	14	42	7	4	15	5	4	42	28..41	
166 (306)	5	15	7	4	15	5	13	10, 12..15	none	[6, (3.7)]
167 (307)	15	25	7	4	15	9	8	none	15..25	
168 (308)	9	15	7	4	15	9	14	9, 12	11, 13..15	
169 (313)	8	30	5	5	15	4	5	27	20..26, 28	[6, (3.7)]
170 (315)	6	18	5	5	15	5	10	18	16, 17	
171 (316)	14	35	5	5	15	6	5	none	24..35	
172 (317)	6	15	5	5	15	6	13	10, 12..15	none	[7, (2.10)], [4]
173 (318)	21	45	5	5	15	7	4	none	35..45	
174 (319)	12	20	5	5	15	9	10	none	15..20	
175 (320)	26	39	5	5	15	10	5	none	26..39	
176 (321)	11	15	5	5	15	11	14	11	13..15	[6, (6.1)]
177 (322)	32	40	5	5	15	12	5	none	27..40	
178 (326)	21	21	5	5	15	15	10	none	21	[1, (3.6)]
179 (327)	4	15	3	6	15	4	11	12, 14, 15	none	[7, (2.8), examples]
180 (328)	13	39	3	6	15	5	4	39	none	
181 (329)	9	27	3	6	15	5	6	27	none	
182 (330)	7	21	3	6	15	5	8	21	none	
183 (331)	5	15	3	6	15	5	12	15	none	
184 (332)	10	25	3	6	15	6	7	none	none	[2]
185 (334)	14	30	3	6	15	7	6	none	28..30	
186 (335)	7	15	3	6	15	7	13	14	none	[7, (2.9)]
187 (336)	13	15	3	6	15	13	14	13	15	[6, (6.1)]
188 (340)	8	15	1	7	15	8	13	14	none	[6, (1.5)], [7, (2.8)]
189 (342)	50	50	1	7	15	15	4	50	none	[7, (2.6)]
190 (344)	15	15	1	7	15	15	14	15	none	[7, (2.6)]

### 1.3 See Also

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[1, 2, 3] For excellent surveys and the original BTD parameter list. An in-progress paper of Billington will update that list.

[6, 7] For some BPTD results and constructions.

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## 6 Balanced (Part) Ternary Designs

### References

- [1] E.J. Billington, Balanced  $n$ -ary designs: A combinatorial survey and some new results, *Ars Combin.*, **17A** (1984), 37–72.
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- [4] P. Chawathe and D.G. Sarvate, Improved entries in a list of BPTDs, ms.
- [5] J.F. Dillon and M.A. Wertheimer, Balanced ternary designs derived from other combinatorial designs, *Congressus Numerantium*, **47** (1985), 285–298.
- [6] T. Kunkle and D.G. Sarvate, On ternary designs with a specified number of blocks with repeated elements, *Ars Combin.*, **40** (1995), 129–142.
- [7] T. Kunkle and D.G. Sarvate, Balanced part ternary designs: some new results, *J. Combin. Math. Combin. Comput.*, to appear.
- [8] K. Sinha, A construction of balanced ternary designs, *Ars Combin.*, **33** (1992), 276–278.