8. $\displaystyle\int_0^{\sqrt{2}}\int_0^{3y}\int_{x^2+3y^2}^{8-x^2-y^2} dz\,dx\,dy$

9. $\displaystyle\int_1^e\int_1^e\int_1^e \frac{1}{xyz}\,dx\,dy\,dz$

10. $\displaystyle\int_0^1\int_0^{3-3x}\int_0^{3-3x-y} dz\,dy\,dx$

11. $\displaystyle\int_0^1\int_0^{\pi}\int_0^{\pi} y\sin z\,dx\,dy\,dz$

12. $\displaystyle\int_{-1}^1\int_{-1}^1\int_{-1}^1 (x+y+z)\,dy\,dx\,dz$

13. $\displaystyle\int_0^3\int_0^{\sqrt{9-x^2}}\int_0^{\sqrt{9-x^2}} dz\,dy\,dx$

14. $\displaystyle\int_0^2\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}}\int_0^{2x+y} dz\,dx\,dy$

15. $\displaystyle\int_0^1\int_0^{2-x}\int_0^{2-x-y} dz\,dy\,dx$

16. $\displaystyle\int_0^1\int_0^{1-x^2}\int_3^{4-x^2-y} x\,dz\,dy\,dx$

17. $\displaystyle\int_0^{\pi}\int_0^{\pi}\int_0^{\pi} \cos(u+v+w)\,du\,dv\,dw$ (uvw-space)

18. $\displaystyle\int_1^e\int_1^e\int_1^e \ln r\ln s\ln t\,dt\,dr\,ds$ (rst-space)

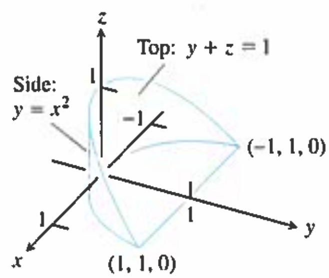
19. $\displaystyle\int_0^{\pi/4}\int_0^{\ln\sec v}\int_{-\infty}^{2t} e^x\,dx\,dt\,dv$ (tvx-space)

20. $\displaystyle\int_0^7\int_0^2\int_0^{\sqrt{4-q^2}} \frac{q}{r+1}\,dp\,dq\,dr$ (pqr-space)

Volumes Using Triple Integrals

21. Here is the region of integration of the integral

$$\int_{-1}^1\int_{x^2}^1\int_0^{1-y} dz\,dy\,dx.$$

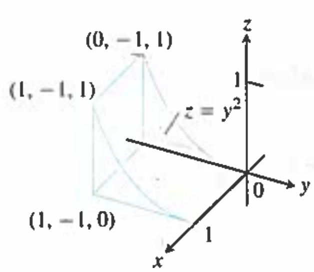


Rewrite the integral as an equivalent iterated integral in the order

a) $dy\,dz\,dx$ b) $dy\,dx\,dz$
c) $dx\,dy\,dz$ d) $dx\,dz\,dy$
e) $dz\,dx\,dy$

22. Here is the region of integration of the integral

$$\int_0^1\int_{-1}^0\int_0^{y^2} dz\,dy\,dx.$$

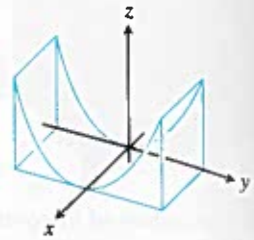


Rewrite the integral as an equivalent iterated integral in the order

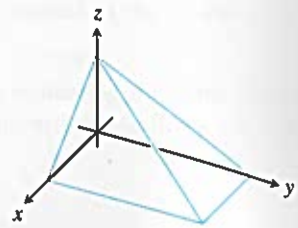
a) $dy\,dz\,dx$ b) $dy\,dx\,dz$
c) $dx\,dy\,dz$ d) $dx\,dz\,dy$
e) $dz\,dx\,dy$

Find the volumes of the regions in Exercises 23–36.

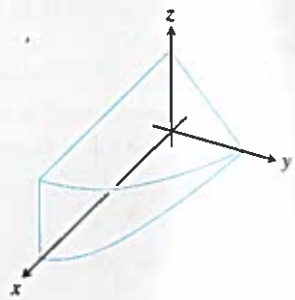
23. The region between the cylinder $z = y^2$ and the xy-plane that is bounded by the planes $x = 0, x = 1, y = -1, y = 1$



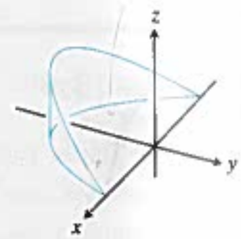
24. The region in the first octant bounded by the coordinate planes and the planes $x + z = 1$, $y + 2z = 2$



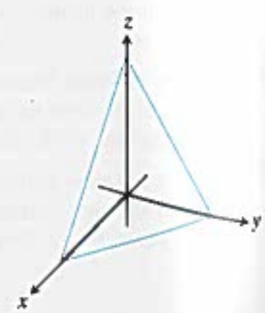
25. The region in the first octant bounded by the coordinate planes, the plane $y + z = 2$, and the cylinder $x = 4 - y^2$



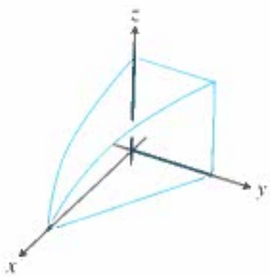
26. The wedge cut from the cylinder $x^2 + y^2 = 1$ by the planes $z = -y$ and $z = 0$



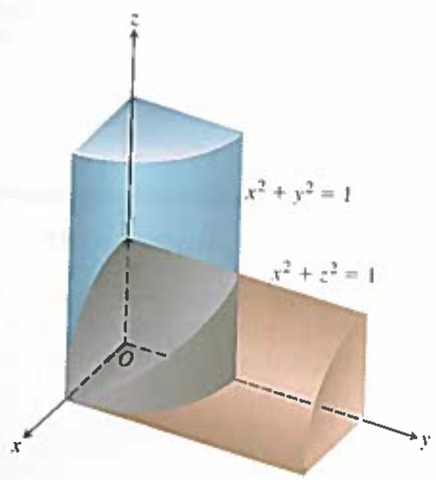
27. The tetrahedron in the first octant bounded by the coordinate planes and the plane $x + y/2 + z/3 = 1$



28. The region in the first octant bounded by the coordinate planes, the plane $y = 1 - x$, and the surface $z = \cos(\pi x/2)$, $0 \le x \le 1$

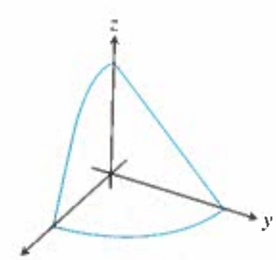


29. The region common to the interiors of the cylinders $x^2 + y^2 = 1$ and $x^2 + z^2 = 1$ (Fig. 13.34)

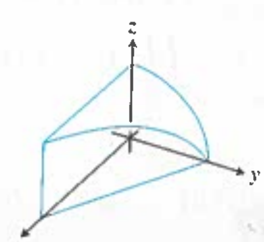


13.34 One-eighth of the region common to the cylinders $x^2 + y^2 = 1$ and $x^2 + z^2 = 1$ in Exercise 29.

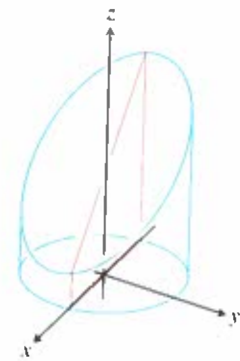
30. The region in the first octant bounded by the coordinate planes and the surface $z = 4 - x^2 - y$



31. The region in the first octant bounded by the coordinate planes, the plane $x + y = 4$, and the cylinder $y^2 + 4z^2 = 16$



32. The region cut from the cylinder $x^2 + y^2 = 4$ by the plane $z = 0$ and the plane $x + z = 3$



33. The region between the planes $x + y + 2z = 2$ and $2x + 2y + z = 4$ in the first octant

34. The finite region bounded by the planes $z = x$, $x + z = 8$, $z = y$, $y = 8$, and $z = 0$.

35. The region cut from the solid elliptical cylinder $x^2 + 4y^2 \le 4$ by the xy-plane and the plane $z = x + 2$

36. The region bounded in back by the plane $x = 0$, on the front and sides by the parabolic cylinder $x = 1 - y^2$, on the top by the paraboloid $z = x^2 + y^2$, and on the bottom by the xy-plane

Average Values

In Exercises 37–40, find the average value of $F(x, y, z)$ over the given region.

37. $F(x, y, z) = x^2 + 9$ over the cube in the first octant bounded by the coordinate planes and the planes $x = 2$, $y = 2$, and $z = 2$

38. $F(x, y, z) = x + y - z$ over the rectangular solid in the first octant bounded by the coordinate planes and the planes $x = 1$, $y = 1$, and $z = 2$

39. $F(x, y, z) = x^2 + y^2 + z^2$ over the cube in the first octant bounded by the coordinate planes and the planes $x = 1$, $y = 1$, and $z = 1$

40. $F(x, y, z) = xyz$ over the cube in the first octant bounded by the coordinate planes and the planes $x = 2$, $y = 2$, and $z = 2$

Changing the Order of Integration

Evaluate the integrals in Exercises 41–44 by changing the order of integration in an appropriate way.

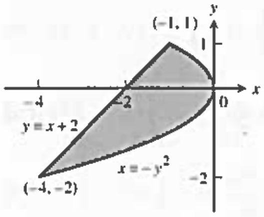
41. $\displaystyle\int_0^4\int_0^1\int_{2y}^2 \frac{4\cos(x^2)}{2\sqrt{z}}\,dx\,dy\,dz$

42. $\displaystyle\int_0^1\int_0^1\int_{x^2}^1 12xze^{zy^2}\,dy\,dx\,dz$

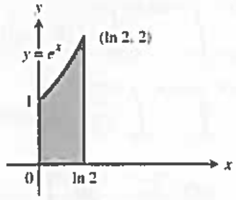
43. $\displaystyle\int_0^1\int_{\sqrt[3]{z}}^1\int_0^{\ln 3} \frac{\pi e^{2x}\sin \pi y^2}{y^2}\,dx\,dy\,dz$

44. $\displaystyle\int_0^2\int_0^{4-x^2}\int_0^x \frac{\sin 2z}{4-z}\,dy\,dz\,dx$

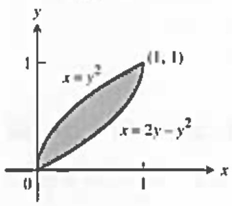
3. $\int_{-2}^{1}\int_{y-2}^{-y^2} dx\,dy = 9/2$



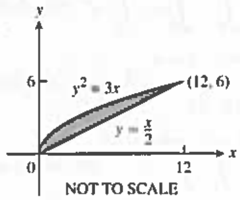
5. $\int_0^{\ln 2}\int_0^{e^x} dy\,dx = 1$



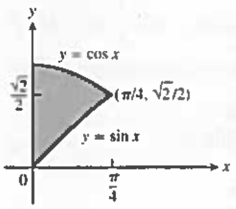
7. $\int_0^1\int_{y^2}^{2y-y^2} dx\,dy = 1/3$



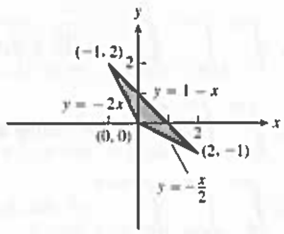
9. 12



11. $\sqrt{2}-1$



13. 3/2



15. a) 0 b) $4/\pi^2$ 17. 8/3 19. $\bar{x}=\frac{5}{14}$, $\bar{y}=\frac{38}{35}$

21. $\bar{x}=\frac{64}{35}$, $\bar{y}=\frac{5}{7}$ 23. $\bar{x}=0$, $\bar{y}=\frac{4}{3\pi}$ 25. $\bar{x}=\bar{y}=\frac{4a}{3\pi}$

27. $\bar{x}=\frac{\pi}{2}$, $\bar{y}=\frac{\pi}{8}$ 29. $\bar{x}=-1$, $\bar{y}=\frac{1}{4}$

31. $I_x=\frac{64}{105}$, $R_x=2\sqrt{\frac{2}{7}}$ 33. $\bar{x}=\frac{3}{8}$, $\bar{y}=\frac{17}{16}$

35. $\bar{x}=\frac{11}{3}$, $\bar{y}=\frac{14}{27}$, $I_y=432$, $R_y=4$

37. $\bar{x}=0$, $\bar{y}=\frac{13}{31}$, $I_y=\frac{7}{5}$, $R_y=\sqrt{\frac{21}{31}}$

39. $\bar{x}=0$, $\bar{y}=7/10$; $I_x=9/10$, $I_y=3/10$, $I_0=6/5$; $R_x=\frac{3\sqrt{6}}{10}$, $R_y=\frac{3\sqrt{2}}{10}$, $R_0=\frac{3\sqrt{2}}{5}$

41. $40{,}000(1-e^{-2})\ln\left(\frac{7}{2}\right)\approx 43{,}329$

43. If $0<a\le 5/2$, then the appliance will have to be tipped more than 45° to fall over.

45. $(\bar{x},\bar{y})=(2/\pi,0)$ 47. a) 3/2 b) They are the same.

53. a) $\left(\frac{7}{5},\frac{31}{10}\right)$ b) $\left(\frac{19}{7},\frac{18}{7}\right)$ c) $\left(\frac{9}{2},\frac{19}{8}\right)$ d) $\left(\frac{11}{4},\frac{43}{16}\right)$

55. In order for c.m. to be on the common boundary, $h=a\sqrt{2}$. In order for c.m. to be inside T, $h>a\sqrt{2}$.

Section 13.3, pp. 1024–1026

1. $\pi/2$ 3. $\pi/8$ 5. πa^2 7. 36 9. $(1-\ln 2)\pi$

11. $(2\ln 2-1)(\pi/2)$ 13. $\frac{\pi}{2}+1$ 15. $\pi(\ln(4)-1)$

17. $2(\pi-1)$ 19. 12π 21. $\frac{3\pi}{8}+1$ 23. 4 25. $6\sqrt{3}-2\pi$

27. $\bar{x}=5/6$, $\bar{y}=0$ 29. $2a/3$ 31. $2a/3$ 33. 2π

35. $\frac{4}{3}+\frac{5\pi}{8}$ 37. a) $\sqrt{\pi}/2$ b) 1 39. $\pi\ln 4$, no

41. $\frac{1}{2}(a^2+2h^2)$

Section 13.4, pp. 1031–1034

1. 1

3. $\int_0^1\int_0^{2-2x}\int_0^{3-3x-3y/2} dz\,dy\,dx$, $\int_0^2\int_0^{1-y/2}\int_0^{3-3x-3y/2} dz\,dx\,dy$,

$\int_0^1\int_0^{3-3x}\int_0^{2-2x-2z/3} dy\,dz\,dx$, $\int_0^3\int_0^{1-z/3}\int_0^{2-2x-2z/3} dy\,dx\,dz$,

$\int_0^2\int_0^{3-3y/2}\int_0^{1-y/2-z/3} dx\,dz\,dy$, $\int_0^3\int_0^{2-2z/3}\int_0^{1-y/2-z/3} dx\,dy\,dz$.

The value of all six integrals is 1.

5. $\int_{-2}^2\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{x^2+y^2}^{8-x^2-y^2} 1\,dz\,dy\,dx$,

$\int_{-2}^2\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}}\int_{x^2+y^2}^{8-x^2-y^2} 1\,dz\,dx\,dy$,

$\int_{-2}^2\int_4^{8-y^2}\int_{-\sqrt{8-z-y^2}}^{\sqrt{8-z-y^2}} 1\,dx\,dz\,dy+\int_{-2}^2\int_{y^2}^4\int_{-\sqrt{z-y^2}}^{\sqrt{z-y^2}} 1\,dx\,dz\,dy$,

$\int_4^8\int_{-\sqrt{8-z}}^{\sqrt{8-z}}\int_{-\sqrt{8-z-y^2}}^{\sqrt{8-z-y^2}} 1\,dx\,dy\,dz+\int_0^4\int_{-\sqrt{z}}^{\sqrt{z}}\int_{-\sqrt{z-y^2}}^{\sqrt{z-y^2}} 1\,dx\,dy\,dz$,

$\int_{-2}^2\int_4^{8-x^2}\int_{-\sqrt{8-z-x^2}}^{\sqrt{8-z-x^2}} 1\,dy\,dz\,dx+\int_{-2}^2\int_{x^2}^4\int_{-\sqrt{z-x^2}}^{\sqrt{z-x^2}} 1\,dy\,dz\,dx$,

$\int_4^8\int_{-\sqrt{8-z}}^{\sqrt{8-z}}\int_{-\sqrt{8-z-x^2}}^{\sqrt{8-z-x^2}} 1\,dy\,dx\,dz+\int_0^4\int_{-\sqrt{z}}^{\sqrt{z}}\int_{-\sqrt{z-x^2}}^{\sqrt{z-x^2}} 1\,dy\,dx\,dz$.

The value of all six integrals is 16π.

7. 1 9. 1 11. $\frac{\pi^3}{2}(1-\cos 1)$ 13. 18 15. 7/6 17. 0

19. $\frac{1}{2}-\frac{\pi}{8}$ 21. a) $\int_{-1}^1\int_0^{1-x^2}\int_{x^2}^{1-z} dy\,dz\,dx$

b) $\int_0^1\int_{-\sqrt{1-z}}^{\sqrt{1-z}}\int_{x^2}^{1-z} dy\,dx\,dz$ c) $\int_0^1\int_0^{1-z}\int_{-\sqrt{y}}^{\sqrt{y}} dy\,dz\,dx$

d) $\int_0^1\int_0^{1-y}\int_{-\sqrt{y}}^{\sqrt{y}} dx\,dz\,dy$ e) $\int_0^1\int_{-\sqrt{y}}^{\sqrt{y}}\int_0^{1-y} dz\,dx\,dy$ **23.** 2/3

25. 20/3 **27.** 1 **29.** 16/3 **31.** $8\pi - \frac{32}{3}$ **33.** 2 **35.** 4π

37. 31/3 **39.** 1 **41.** $2\sin 4$ **43.** 4 **45.** $a = 3$ or $a = \frac{13}{3}$

Section 13.5, pp. 1036–1039

1. $R_x = \sqrt{\frac{b^2+c^2}{12}}$, $R_y = \sqrt{\frac{a^2+c^2}{12}}$, $R_z = \sqrt{\frac{a^2+b^2}{12}}$

3. $I_x = \frac{M}{3}(b^2+c^2)$, $I_y = \frac{M}{3}(a^2+c^2)$, $I_z = \frac{M}{3}(a^2+b^2)$

5. $\bar{x} = \bar{y} = 0$, $\bar{z} = \frac{12}{5}$, $I_x = \frac{7904}{105} \approx 75.28$, $I_y = \frac{4832}{63} \approx 76.70$, $I_z = \frac{256}{45} \approx 5.69$

7. a) $\bar{x} = \bar{y} = 0$, $\bar{z} = \frac{8}{3}$ b) $c = 2\sqrt{2}$

9. $I_L = 1386$, $R_L = \sqrt{\frac{77}{2}}$ **11.** $I_L = \frac{40}{3}$, $R_L = \sqrt{\frac{5}{3}}$ **13.** a) $\frac{4}{3}$

b) $\bar{x} = \frac{4}{5}$, $\bar{y} = \bar{z} = \frac{2}{5}$ **15.** a) $\frac{5}{2}$ b) $\bar{x} = \bar{y} = \bar{z} = \frac{8}{15}$

c) $I_x = I_y = I_z = \frac{11}{6}$ d) $R_x = R_y = R_z = \sqrt{\frac{11}{15}}$ **17.** 3

19. a) $\frac{4}{3}g$ b) $\frac{4}{3}g$

23. a) $I_{c.m.} = \frac{abc(a^2+b^2)}{12}$, $R_{c.m.} = \sqrt{\frac{a^2+b^2}{12}}$

b) $I_L = \frac{abc(a^2+7b^2)}{3}$, $R_L = \sqrt{\frac{a^2+7b^2}{3}}$

27. a) $h = a\sqrt{3}$ b) $h = a\sqrt{2}$

Section 13.6, pp. 1044–1047

1. $4\pi(\sqrt{2}-1)/3$ **3.** $17\pi/5$ **5.** $\pi(6\sqrt{2}-8)$ **7.** $3\pi/10$

9. $\pi/3$ **11.** a) $\int_0^{2\pi}\int_0^1\int_0^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta$

b) $\int_0^{2\pi}\int_0^{\sqrt{3}}\int_0^1 r\,dr\,dz\,d\theta + \int_0^{2\pi}\int_{\sqrt{3}}^2\int_0^{\sqrt{4-z^2}} r\,dr\,dz\,d\theta$

c) $\int_0^1\int_0^{\sqrt{4-r^2}}\int_0^{2\pi} r\,d\theta\,dz\,dr$

13. $\int_{-\pi/2}^{\pi/2}\int_0^{\cos\theta}\int_0^{3r^2} f(r,\theta,z)\,r\,dz\,dr\,d\theta$

15. $\int_0^{\pi}\int_0^{2\sin\theta}\int_0^{4-r\sin\theta} f(r,\theta,z)\,dz\,r\,dr\,d\theta$

17. $\int_{-\pi/2}^{\pi/2}\int_1^{1+\cos\theta}\int_0^4 f(r,\theta,z)\,dz\,r\,dr\,d\theta$

19. $\int_0^{\pi/4}\int_0^{\sec\theta}\int_0^{2-r\sin\theta} f(r,\theta,z)\,dz\,r\,dr\,d\theta$ **21.** π^2 **23.** $\pi/3$

25. 5π **27.** 2π **29.** $\left(\frac{8-5\sqrt{2}}{2}\right)\pi$

31. a) $\int_0^{2\pi}\int_0^{\pi/6}\int_0^2 \rho^2\sin\phi\,d\rho\,d\phi\,d\theta + \int_0^{2\pi}\int_{\pi/6}^{\pi/2}\int_0^{\csc\phi} \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

b) $\int_0^{2\pi}\int_1^2\int_{\pi/6}^{\sin^{-1}(1/\rho)} \rho^2\sin\phi\,d\phi\,d\rho\,d\theta + \int_0^{2\pi}\int_0^2\int_0^{\pi/6} \rho^2\sin\phi\,d\phi\,d\rho\,d\theta$

33. $\int_0^{2\pi}\int_0^{\pi/2}\int_{\cos\phi}^2 \rho^2\sin\phi\,d\rho\,d\phi\,d\theta = \frac{31\pi}{6}$

35. $\int_0^{2\pi}\int_0^{\pi}\int_0^{1-\cos\phi} \rho^2\sin\phi\,d\rho\,d\phi\,d\theta = \frac{8\pi}{3}$

37. $\int_0^{2\pi}\int_{\pi/4}^{\pi/2}\int_0^{2\cos\phi} \rho^2\sin\phi\,d\rho\,d\phi\,d\theta = \frac{\pi}{3}$

39. a) $8\int_0^{\pi/2}\int_0^{\pi/2}\int_0^2 \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

b) $8\int_0^{\pi/2}\int_0^2\int_0^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta$

c) $8\int_0^2\int_0^{\sqrt{4-x^2}}\int_0^{\sqrt{4-x^2-y^2}} dz\,dy\,dx$

41. a) $\int_0^{2\pi}\int_0^{\pi/3}\int_{\sec\phi}^2 \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

b) $\int_0^{2\pi}\int_0^{\sqrt{3}}\int_1^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta$

c) $\int_{-\sqrt{3}}^{\sqrt{3}}\int_{-\sqrt{3-x^2}}^{\sqrt{3-x^2}}\int_1^{\sqrt{4-x^2-y^2}} dz\,dy\,dx$ d) $\frac{5\pi}{3}$ **43.** $8\pi/3$

45. 9/4 **47.** $(3\pi-4)/18$ **49.** $\frac{2\pi a^3}{3}$ **51.** $5\pi/3$ **53.** $\pi/2$

55. $\frac{4(2\sqrt{2}-1)\pi}{3}$ **57.** 16π **59.** $5\pi/2$ **61.** $\frac{4\pi(8-3\sqrt{3})}{3}$

63. 2/3 **65.** 3/4 **67.** $\bar{x} = \bar{y} = 0$, $\bar{z} = 3/8$

69. $(\bar{x}, \bar{y}, \bar{z}) = (0, 0, 3/8)$ **71.** $\bar{x} = \bar{y} = 0$, $\bar{z} = 5/6$