1 (10 pts). Find the volume of the solid beneath the surface \( z = \frac{1}{x^2 + y^2} \) and above the annulus (ring) \( 1 \leq x^2 + y^2 \leq 2 \) in the \( xy \)-plane.

**Solution:** 1. (Source: 15.3.20) The domain of integration is the region inside the circle of radius \( \sqrt{2} \) and outside the circle of radius 1, both centered at the origin. Both the integrand and the region of integration are most easily described in polar coordinates. The volume equals

\[
\int_0^{2\pi} \int_1^{\sqrt{2}} \frac{1}{r^2} r \, dr \, d\theta = \int_0^{2\pi} \int_1^{\sqrt{2}} \frac{1}{r} \, dr \, d\theta = \int_0^{2\pi} \ln r \bigg|_1^{\sqrt{2}} \, d\theta
\]

The integral of a constant is just the constant times the length of the interval of integration, so this equals

\[
2\pi \left( \ln (2^{1/2}) - \ln 1 \right) = \pi \ln 2.
\]

(done)