MATH 221–02 (Kunkle), Quiz 3 | Name: ____________________
10 pts, 10 minutes | Jan. 30, 2017

---

1 (10 pts). Let $\mathbf{r}(t) = \langle 2t, t^2, e^{-t} \rangle$ be the position of a particle at time t.

a. Find the object's velocity, acceleration, speed, and the normal and tangential components of acceleration at time $t = 0$.

b. Find equations of the normal and osculating planes to the object's path at the point corresponding to time $t = 0$.

Solution: 1a.(Source: 13.4.41)

$$\begin{aligned} \text{position} &= \mathbf{r}(t) = \langle 2t, t^2, e^{-t} \rangle \\ \text{velocity} &= \mathbf{v}(t) = \frac{d\mathbf{r}}{dt} = \langle 2, 2t, -e^{-t} \rangle \\ \text{acceleration} &= \mathbf{a}(t) = \frac{d\mathbf{v}}{dt} = \langle 0, 2, e^{-t} \rangle \end{aligned}$$

Evaluate these at $t = 0$

$$\begin{aligned} \mathbf{r}(0) &= \langle 0, 0, 1 \rangle \\ \mathbf{v}(0) &= \langle 2, 0, -1 \rangle \\ \mathbf{a}(0) &= \langle 0, 2, 1 \rangle \end{aligned}$$

and then calculate

$$\begin{aligned} \text{speed} = |\mathbf{v}| &= \sqrt{5} \\ a_T &= \frac{\mathbf{v} \cdot \mathbf{a}}{|\mathbf{v}|} = \frac{-1}{\sqrt{5}} \\ a_N &= \frac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|} = \frac{|2\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}|}{\sqrt{5}} = \sqrt{\frac{24}{5}} \end{aligned}$$

For this last line, I calculated the cross product

$$\mathbf{v} \times \mathbf{a} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0 & -1 \\ 0 & 2 & 1 \end{vmatrix} = \langle 2, -2, 4 \rangle.$$

1b.(Source: 13.3.50) The normal plane passes through the point $(0, 0, 1)$ and is normal to $\mathbf{v} = \langle 2, 0, -1 \rangle$. Its equation is

$$2x - (z - 1) = 0.$$

The osculating plane passes through $(0, 0, 1)$ and is normal to $\mathbf{v} \times \mathbf{a} = \langle 2, -2, 4 \rangle$. Its equation is

$$2x - 2y + 4(z - 1) = 0,$$

or, more simply,

$$x - y + 2(z - 1) = 0.$$