MATH 221–02 (Kunkle), Quiz 1 | Name: ______________________________
10 pts, 10 minutes | Jan. 19, 2017

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1 (10 pts). Find an equation of the plane through the three points $(1, 1, 0)$, $(1, -2, 3)$, and $(5, 4, 2)$.

Solution: 1(Source: 12.5.31-34) . The vectors

$$\langle 1, -2, 3\rangle - \langle 1, 1, 0\rangle = \langle 0, -3, 3\rangle$$

and

$$\langle 5, 4, 2\rangle - \langle 1, 1, 0\rangle = \langle 4, 3, 2\rangle$$

are parallel the plane, so the plane is orthogonal to their cross product:

$$\begin{aligned}\langle 0, -3, 3\rangle \times \langle 4, 3, 2\rangle &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -3 & 3 \\ 4 & 3 & 2 \end{vmatrix} \\ &= 3\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -1 & 1 \\ 4 & 3 & 2 \end{vmatrix} \\ &= 3\left(\mathbf{i}\begin{vmatrix} -1 & 1 \\ 3 & 2 \end{vmatrix} - \mathbf{j}\begin{vmatrix} 0 & 1 \\ 4 & 2 \end{vmatrix} + \mathbf{k}\begin{vmatrix} 0 & -1 \\ 4 & 3 \end{vmatrix}\right) \\ &= 3(-5\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}).\end{aligned}$$

The plane must also be orthogonal to $-5\mathbf{i}+4\mathbf{j}+4\mathbf{k}$, so we'll use this for the normal vector. Then the plane has the equation

$$-5(x-1) + 4(y-1) + 4z = 0,$$

or, if you prefer, $-5x + 4y + 4z = -1$. (See page 827.) **(done)**

Tip: A plane always has an equation of the form

$$Ax + By + Cz = D$$

and, conversely, the graph of any such equation is a plane. Don't confuse the equation of a plane with the parametric equations of a line.