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1a (8 pts). Find a parametric equation of the line tangent to the graph of

$$\mathbf{r}(t) = \sin t \mathbf{i} - 3 \cos t \mathbf{j} + t^2 \mathbf{k}$$

at the point corresponding to  $t = \pi$ .

1b (2 pts). Find the unit tangent  $\mathbf{T}$  at the point corresponding to  $t = \pi$  along the same curve.

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*Solution:* 1. Differentiate:

$$\begin{aligned}\mathbf{r}(t) &= \sin t \mathbf{i} - 3 \cos t \mathbf{j} + t^2 \mathbf{k} \\ \frac{d\mathbf{r}}{dt} &= \cos t \mathbf{i} + 3 \sin t \mathbf{j} + 2t \mathbf{k}\end{aligned}$$

At  $t = \pi$ ,

$$\begin{aligned}\mathbf{r} &= 3 \mathbf{j} + \pi^2 \mathbf{k} = \langle 0, 3, \pi^2 \rangle \\ \frac{d\mathbf{r}}{dt} &= -\mathbf{i} + 2\pi \mathbf{k} = \langle -1, 0, 2\pi \rangle\end{aligned}$$

Now, as in section 12.5, the tangent line required in part a is

$$x = -t \quad y = 3 \quad z = \pi^2 + 2\pi t.$$

(Compare with problem 12.5.2, for example.)

Normalize  $\frac{d\mathbf{r}}{dt}$  to find the the unit tangent required in part b:

$$\mathbf{T} = \frac{\langle -1, 0, 2\pi \rangle}{|\langle -1, 0, 2\pi \rangle|} = \frac{1}{\sqrt{1 + 4\pi^2}} \langle -1, 0, 2\pi \rangle.$$

(done)