No notes, books, electronic devices, or outside materials of any kind.

Read each problem carefully and simplify your answers.

Unless otherwise indicated, supporting work will be required on every problem worth more than 2 points.

1(5 pts). Sketch the domain of $f(x, y) = \sqrt{9 - x^2 - y^2}$.
(You might wish to explain in words what you're trying to sketch.)

2(10 pts). Find an equation of the plane tangent to the surface $xy - yz = 8$ at $(1, 2, -3)$.

3. Let $g(x, y) = \tan^{-1}(x^2 y)$.

a(4 pts). Find the first partial derivatives of g.

b(2 pts). Find the gradient of g at the point $(1, 2)$.

c(3 pts). Find $D_{\mathbf{u}}g$ at $(1, 2)$, where $\mathbf{u}$ is the vector $\frac{3}{5}\mathbf{i} - \frac{4}{5}\mathbf{j}$.

d(4 pts). In what direction is g increasing most rapidly at the point $(1, 2)$? Give your answer in the form of a unit vector.

e(6 pts). Find an equation of the plane tangent to the graph of g at the point $(1, 2)$.

f(2 pts). What is the linearization of g at $(1, 2)$?

4(8 pts). Suppose $f(x, y)$ is a differentiable function of x and y. Use the table of values to find $g_u(0, 0)$ and $g_v(0, 0)$ if $g(u, v) = f(4u + 5v, 1 + 2u - 3v)$.

	f	g	f_x	f_y
$(0, 0)$	2	-3	4	-5
$(0, 1)$	-3	2	-1	7

5(8 pts). One of the limits below exists, and the other does not. **Choose one**, and then evaluate that limit or show that it does not exist. Be sure to state clearly which limit you've chosen to work on.

a. $\lim_{(x,y)\to(0,0)} \frac{x^4 - y^2}{x^2 + y}$ b. $\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^4 + y^2}$

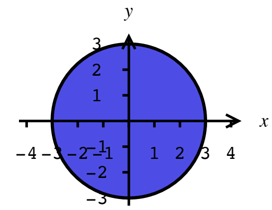
6(16 pts). Find the points in the xy-plane where $h(x, y) = x^3 - y^3 + xy$ has a local max, a local min, or a saddle point and clearly identify which is which. You do not need to report the values of h at those points.

7(16 pts). Find the maximum and minimum values of xy along the curve $x^2 + 2y^2 = 1$ and state at which points the max and min occur.

8(6 pts). Evaluate the double integral $\iint_R (y - xy^{-2})\, dA$ where $R = [0, 2] \times [1, 2]$.

9(10 pts). Estimate the volume of the solid that lies below the surface $z = 1 + x + 2y$ and above the rectangle $[-1, 1] \times [0, 1]$ with a Riemann sum using the Midpoint Rule and $m = n = 2$ subintervals in both the x- and y-direction.
Your answer should contain only numbers and arithmetic operations. You can leave unfinished arithmetic in your answer, but it must specifically show how to calculate the Riemann sum.

1.(Source: 14.1.15,16,31) $\sqrt{9-x^2-y^2}$ is defined on the solution set of $0 \le 9-x^2-y^2$, or $x^2+y^2 \le 9$. This is the circle centered at $(0,0)$ of radius 3 and its interior.



2.(Source: 14.6.41) The surface in question is a level surface of $f(x,y,z) = xy - yz$ and is always perpendicular to $\nabla f = \langle y, x-z, -y\rangle$, so we use $\nabla f(1,2,-3) = \langle 2,4,-2\rangle$, for the normal vector of the plane. Its equation is $2(x-1)+4(y-2)-2(z+3) = 0$, or, more simply, $(x-1)+2(y-2)-(z+3) = 0$.

3a.(Source: 14.3.27) $g_x = \frac{2xy}{1+(x^2y)^2}$, and $g_y = \frac{x^2}{1+(x^2y)^2}$

3b.(Source: 14.6.7-10) $\nabla g(x,y) = \langle g_x(x,y), g_y(x,y)\rangle$. At $(1,2)$, $\nabla g(1,2) = \langle \frac{4}{5}, \frac{1}{5}\rangle$.

3c.(Source: 14.6.11-14) $D_{\mathbf{u}}g(1,2) = \nabla g(1,2)\cdot \mathbf{u} = \langle \frac{4}{5}, \frac{1}{5}\rangle \cdot \langle \frac{3}{5}, -\frac{4}{5}\rangle = \frac{8}{25}$.

3d.(Source: 14.6.26) At $(1,2)$, g increases most rapidly in the direction of its gradient. Tip: to normalize $\langle \frac{4}{5}, \frac{1}{5}\rangle$ is the same as normalizing $\langle 4,1\rangle$, since this is positive multiple of the former. The result is $\frac{1}{\sqrt{17}}\langle 4,1\rangle$.

3e.(Source: 14.4.1-6) The tangent plane is $z = g(1,2) + g_x(1,2)(x-1) + g_y(1,2)(y-2)$, or $z = \tan^{-1}(2) + \frac{4}{5}(x-1) + \frac{1}{5}(y-2)$.

3f.(Source: 14.4.15) From 3e, $L(x,y) = \tan^{-1}(2) + \frac{4}{5}(x-1) + \frac{1}{5}(y-2)$.

4.(Source: 14.5.15,16) Let $x(u,v) = 4u+5v$ and $y(u,v) = 1+2u-3v$. When $(u,v) = (0,0,$ $(x,y) = (0,1)$, and by the Chain Rule,

$$g_u(0,0) = f_x(0,1)x_u(0,0) + f_y(0,1)y_u(0,0) = -1(4) + 7(2) = 10,$$

and

$$g_v(0,0) = f_x(0,1)x_v(0,0) + f_y(0,1)y_v(0,0) = -1(5) + 7(-3) = -26.$$

5a.(Source: 14.2.17) We can take this limit by first simplifying the quotient.

$$\lim_{(x,y)\to(0,0)} \frac{x^4-y^2}{x^2+y} = \lim_{(x,y)\to(0,0)} \frac{(x^2-y)(x^2+y)}{x^2+y} = \lim_{(x,y)\to(0,0)} (x^2-y) = 0.$$

5b.(Source: 14.2.18) As $(x,y)\to(0,0)$ along the x-axis, $\frac{x^2y}{x^4+y^2} = 0$, so the limit can only be 0. But as $(x,y)\to(0,0)$ along the path $y = x^2$, the quotient is $\frac{x^2x^2}{x^4+x^4} = \frac{1}{2}$, so the limit can only be $\frac{1}{2}$. Therefore, the limit does not exist.

6.(Source: 14.7.3) First find all critical points. These are the solutions to

$$h_x(x,y) = 3x^2 + y = 0 \quad \text{and} \quad h_y(x,y) = -3y^2 + x = 0.$$

From the first, $y = -3x^2$, which we substitute into the second:

$$-27x^4 + x = x(-27x^3+1) = 0$$

Either $x = 0$ or $x^3 = \frac{1}{27}$ and therefore $x = \frac{1}{3}$. Find the corresponding y-values from $y = -3x^2$. The critical points are $(0,0)$ and $(\frac{1}{3}, -\frac{1}{3})$.

Now apply the second derivative test.

$$D = \begin{vmatrix} h_{xx} & h_{xy} \\ h_{yx} & h_{yy} \end{vmatrix} = \begin{vmatrix} 6x & 1 \\ 1 & -6y \end{vmatrix} = -36xy - 1.$$

$D(0,0) = -1 < 0$, so h has a saddle point at $(0,0)$. $D(\frac{1}{3}, -\frac{1}{3}) = 4-1 > 0$, and $h_{xx}(\frac{1}{3}, -\frac{1}{3}) = 2 > 0$, so h has a relative minimum at $(\frac{1}{3}, -\frac{1}{3})$.

7.(Source: 14.8.5) Using Lagrange multipliers, we search for those points (x, y) along the contraint at which $\nabla xy = \lambda \nabla(x^2 + 2y^2)$. That is,

$$y = \lambda 2x \qquad x = \lambda 4y \qquad \text{and} \qquad x^2 + 2y^2 = 1$$

If x or y were zero, the first two equations would imply that both equal zero, contradicting the third. So, we can solve for λ in the first two:

$$\lambda = \frac{y}{2x} = \frac{x}{4y}.$$

Cross multiplying,

$$2y^2 = x^2,$$

and so the constraint implies $4y^2 = 1$, or $y = \pm\frac{1}{2}$, and, independent of whether y is positive or negative, $2x^2 = 1$, so $x = \pm\frac{1}{\sqrt{2}}$. Now evaluate xy at the four critical points:

(x, y)	$(\frac{1}{\sqrt{2}}, \frac{1}{2})$	$(-\frac{1}{\sqrt{2}}, \frac{1}{2})$	$(\frac{1}{\sqrt{2}}, -\frac{1}{2})$	$(-\frac{1}{\sqrt{2}}, -\frac{1}{2})$
xy	$\frac{1}{2\sqrt{2}}$	$-\frac{1}{2\sqrt{2}}$	$-\frac{1}{2\sqrt{2}}$	$\frac{1}{2\sqrt{2}}$

The maximum of xy along the constraint is $\frac{1}{2\sqrt{2}}$, and the minimum is $-\frac{1}{2\sqrt{2}}$.

8.(Source: 15.1.28) Evaluate by iterated integration:

$$\int_0^2\int_1^2 (y - xy^{-2})\,dy\,dx = \int_0^2 \left(\tfrac{1}{2}y^2 + xy^{-1}\right)\Big|_1^2 dx = \int_0^2 \left((2 + \tfrac{1}{2}x) - (\tfrac{1}{2} + x)\right) dx$$
$$= \int_0^2 \left(\tfrac{3}{2} - \tfrac{1}{2}x\right) dx = \tfrac{1}{2}(3x - \tfrac{1}{2}x^2)\Big|_0^2 = 2.$$

9.(Source: 15.1.4) The midpoints of the four rectangles are at $x = \pm\frac{1}{2}$ and $y = \frac{1}{4}, \frac{3}{4}$. The Riemann sum is

$$\left(f(-\tfrac{1}{2}, \tfrac{1}{4}) + f(-\tfrac{1}{2}, \tfrac{3}{4}) + f(\tfrac{1}{2}, \tfrac{1}{4}) + f(\tfrac{1}{2}, \tfrac{3}{4})\right)\Delta x \Delta y$$
$$= \left((1 - \tfrac{1}{2} + \tfrac{1}{2}) + (1 - \tfrac{1}{2} + \tfrac{3}{2}) + (1 + \tfrac{1}{2} + \tfrac{1}{2}) + (1 + \tfrac{1}{2} + \tfrac{3}{2})\right)1 \cdot \tfrac{1}{2}.$$

This equals 4, although you were not required to complete that calculation. As a matter of fact, because the integrand is linear, the midpoint rule has calculated the double integral $\iint_R (1 + x + 2y)\,dA$ exactly.