1 (10 pts). Find the derivative of

\[ f(x, y) = \ln \left( \frac{1 + x}{1 - y} \right) \]

at the point \((2, -3)\) in the direction of \(u = \langle 3/5, -4/5 \rangle\).

Solution:

It will help to rewrite \(f(x, y) = \ln(1 + x) - \ln(1 - y)\). Then

\[ \nabla f(x, y) = \left\langle \frac{1}{1 + x}, \frac{1}{1 - y} \right\rangle. \]

At \((2, -3)\), the gradient is \(\nabla f(2, -3) = \langle \frac{1}{3}, \frac{1}{4} \rangle\), so the directional derivative is

\[ D_u f(2, -3) = \left\langle \frac{1}{3}, \frac{1}{4} \right\rangle \cdot \left\langle \frac{3}{5}, \frac{-4}{5} \right\rangle = \frac{1}{5} - \frac{1}{5} = 0. \]

(done)