1 (10 pts). Find the velocity $v(t)$ and position $r(t)$ vectors that correspond to the given acceleration and initial velocity and position.

$$a(t) = e^t i + \cos(t) j + k; \quad v(0) = i + j - k \quad r(0) = -j + k$$

**Solution:**

1. To find velocity, integrate acceleration.

$$v = \int a \, dt = \int \langle e^t, \cos t, 1 \rangle \, dt$$

$$= \langle e^t, \sin t, t \rangle + C$$

Evaluate at $t = 0$ to find the constant (vector) of integration:

$$v(0) = \langle 1, 1, -1 \rangle = \langle e^0, \sin 0, 0 \rangle + C$$

$$= \langle 1, 0, 0 \rangle + C$$

Solving, $C = \langle 0, 1, -1 \rangle$ and

$$v = \langle e^t, 1 + \sin t, t - 1 \rangle$$

Now integrate again to find $r$.

$$r = \int v \, dt = \int \langle e^t, 1 + \sin t, t - 1 \rangle \, dt$$

$$= \langle e^t, t - \cos t, \frac{1}{2} t^2 - t \rangle + C$$

Evaluate at $t = 0$:

$$r(0) = \langle 0, -1, 1 \rangle = \langle e^0, -\cos 0, 0 \rangle + C$$

$$= \langle 1, -1, 0 \rangle + C$$

Solving, $C = \langle -1, 0, 1 \rangle$ and

$$r = \langle e^t - 1, t - \cos t, \frac{1}{2} t^2 - t + 1 \rangle$$

(done)