1. This problem is about the graph of the equation $4x^2 - 2y^2 + z^2 = 4$.

a (2 pts). Find the coordinates of all intercepts of this surface with the coordinate axes.

b (3 pts). Complete the following statements by choosing the correct word.
   - Cross-sections at $x = \text{constant}$ (when they exist) are \textbf{hyperbolas}.
   - Cross-sections at $y = \text{constant}$ (when they exist) are \textbf{ellipses}.
   - Cross-sections at $z = \text{constant}$ (when they exist) are \textbf{hyperbolas}.

c (5 pts). Make a sketch of the surface.

d (1 pt). Extra Credit: what is the name for such a surface?

\textbf{Solution:}

1.a. To find the intercepts, set two of the variables equal zero and solve for the third.

   \begin{align*}
   x &= y = 0 \\
   y &= z = 0 \\
   x &= z = 0 \\
   z^2 &= 4 \\
   4x^2 &= 4 \\
   -2y^2 &= 4 \\
   z &= \pm 2 \\
   x &= \pm 1 \\
   \text{no real solutions}
   \end{align*}

   \begin{align*}
   \text{intercepts:} & \quad \text{intercepts:} & \quad \text{no intercepts} \\
   (0, 0, 2), (0, 0, -2) & \quad (1, 0, 0), (-1, 0, 0)
   \end{align*}

b. At $x = C$, equation becomes $-2y^2 + z^2 = C$. Cross-sections are \textbf{hyperbolas}.

At $y = C$, equation becomes $4x^2 + z^2 = \text{constant}$. Cross-sections are \textbf{ellipses}.

At $z = C$, equation becomes $4x^2 - 2y^2 = \text{constant}$. Cross-sections are \textbf{hyperbolas}.

c. Figure II on page 811 shows such a surface. Here’s a drawing by hand:

\begin{center}
\includegraphics[width=0.5\textwidth]{hyperboloid.png}
\end{center}

d. This is a \textbf{hyperboloid of one sheet}. 