

More problems for section 7.2 of *Calculus, Early Transcendentals* by James Stewart, 8e.

1. Derive the reduction formulas. Hint: use integration by parts only in a, b, and c. See example 6 in §7.1.

a.
$$\int \sin^n x \, dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x \, dx$$

b.
$$\int \cos^n x \, dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x \, dx$$

c.
$$\int \sec^n x \, dx = \frac{1}{n-1} \sec^{n-2} x \tan x + \frac{n-2}{n-1} \int \sec^{n-2} x \, dx$$

d.
$$\int \tan^n x \, dx = \frac{1}{n-1} \tan^{n-1} x - \int \tan^{n-2} x \, dx$$

2. Integrate. Hint: first look for those rows or columns that can be solved by a single method. Reduction formulas do not always provide the fastest solution.

a. $\int dx$ b. $\int \tan x \, dx$ c. $\int \tan^2 x \, dx$ d. $\int \tan^3 x \, dx$ e. $\int \tan^4 x \, dx$

f. $\int \sec x \, dx$ g. $\int \tan x \sec x \, dx$ h. $\int \tan^2 x \sec x \, dx$ i. $\int \tan^3 x \sec x \, dx$ j. $\int \tan^4 x \sec x \, dx$

k. $\int \sec^2 x \, dx$ l. $\int \tan x \sec^2 x \, dx$ m. $\int \tan^2 x \sec^2 x \, dx$ n. $\int \tan^3 x \sec^2 x \, dx$ o. $\int \tan^4 x \sec^2 x \, dx$

p. $\int \sec^3 x \, dx$ q. $\int \tan x \sec^3 x \, dx$ r. $\int \tan^2 x \sec^3 x \, dx$ s. $\int \tan^3 x \sec^3 x \, dx$ t. $\int \tan^4 x \sec^3 x \, dx$

u. $\int \sec^4 x \, dx$ v. $\int \tan x \sec^4 x \, dx$ w. $\int \tan^2 x \sec^4 x \, dx$ x. $\int \tan^3 x \sec^4 x \, dx$ y. $\int \tan^4 x \sec^4 x \, dx$

z. $\int \sec^5 x \, dx$ 27. $\int \tan x \sec^5 x \, dx$ 28. $\int \tan^2 x \sec^5 x \, dx$ 29. $\int \tan^3 x \sec^5 x \, dx$ 30. $\int \tan^4 x \sec^5 x \, dx$

31. $\int \sec^6 x \, dx$ 32. $\int \tan x \sec^6 x \, dx$ 33. $\int \tan^2 x \sec^6 x \, dx$ 34. $\int \tan^3 x \sec^6 x \, dx$ 35. $\int \tan^4 x \sec^6 x \, dx$

3. Integrate. Hint: first look for those rows or columns that can be solved by a single method. Reduction formulas do not always provide the fastest solution.

- a. $\int dx$ b. $\int \cos x dx$ c. $\int \cos^2 x dx$ d. $\int \cos^3 x dx$ e. $\int \cos^4 x dx$
- f. $\int \sin x dx$ g. $\int \sin x \cos x dx$ h. $\int \sin x \cos^2 x dx$ i. $\int \sin x \cos^3 x dx$ j. $\int \sin x \cos^4 x dx$
- k. $\int \sin^2 x dx$ l. $\int \sin^2 x \cos x dx$ m. $\int \sin^2 x \cos^2 x dx$ n. $\int \sin^2 x \cos^3 x dx$ o. $\int \sin^2 x \cos^4 x dx$
- p. $\int \sin^3 x dx$ q. $\int \sin^3 x \cos x dx$ r. $\int \sin^3 x \cos^2 x dx$ s. $\int \sin^3 x \cos^3 x dx$ t. $\int \sin^3 x \cos^4 x dx$
- u. $\int \sin^4 x dx$ v. $\int \sin^4 x \cos x dx$ w. $\int \sin^4 x \cos^2 x dx$ x. $\int \sin^4 x \cos^3 x dx$ y. $\int \sin^4 x \cos^4 x dx$
- z. $\int \sin^5 x dx$ 27. $\int \sin^5 x \cos x dx$ 28. $\int \sin^5 x \cos^2 x dx$ 29. $\int \sin^5 x \cos^3 x dx$ 30. $\int \sin^5 x \cos^4 x dx$
31. $\int \sin^6 x dx$ 32. $\int \sin^6 x \cos x dx$ 33. $\int \sin^6 x \cos^2 x dx$ 34. $\int \sin^6 x \cos^3 x dx$ 35. $\int \sin^6 x \cos^4 x dx$

Answers

- 2a. $x + C$ 2b. $\ln |\sec x| + C$ 2c. $\tan x - x + C$ 2d. $\frac{1}{2} \tan^2 x - \ln |\sec x| + C$ 2e. $\frac{1}{3} \tan^3 x - \tan x + x + C$ 2f. $\ln |\sec x + \tan x| + C$
 2g. $\sec x + C$ 2h. $\frac{1}{2} \sec x \tan x - \frac{1}{2} \ln |\sec x + \tan x| + C$ 2i. $\frac{1}{3} \sec^3 x - \sec x + C$ 2j. $\frac{1}{4} \sec^3 x \tan x - \frac{5}{8} \sec x \tan x + \frac{3}{8} \ln |\sec x + \tan x| + C$
 2k. $\tan x + C$ 2l. $\frac{1}{2} \tan^2 x + C$ 2m. $\frac{1}{3} \tan^3 x + C$ 2n. $\frac{1}{4} \tan^4 x + C$ 2o. $\frac{1}{5} \tan^5 x + C$ 2p. $\frac{1}{2} \sec x \tan x + \frac{1}{2} \ln |\sec x + \tan x| + C$
 2q. $\frac{1}{3} \sec^3 x + C$ 2r. $\frac{1}{4} \sec^3 x \tan x + \frac{1}{8} \sec x \tan x + \frac{1}{8} \ln |\sec x + \tan x| + C$ 2s. $\frac{1}{5} \sec^5 x - \frac{1}{3} \sec^3 x + C$ 2t. $\frac{1}{6} \sec^5 x \tan x - \frac{7}{24} \sec^3 x \tan x + \frac{1}{16} \sec x \tan x + \frac{1}{16} \ln |\sec x + \tan x| + C$ 2u. $\tan x + \frac{1}{3} \tan^3 x + C$ 2v. $\frac{1}{2} \tan^2 x + \frac{1}{4} \tan^4 x + C$ 2w. $\frac{1}{5} \tan^3 x + \frac{1}{5} \tan^5 x + C$
 2x. $\frac{1}{4} \tan^4 x + \frac{1}{6} \tan^6 x + C$ 2y. $\frac{1}{5} \tan^5 x + \frac{1}{7} \tan^7 x + C$ 2z. $\frac{1}{4} \sec^3 x \tan x + \frac{3}{8} \sec x \tan x + \frac{3}{8} \ln |\sec x + \tan x| + C$ 227. $\frac{1}{5} \sec^5 x + C$
 228. $\frac{1}{6} \sec^5 x \tan x - \frac{1}{24} \sec^3 x \tan x - \frac{1}{16} \sec x \tan x - \frac{1}{16} \ln |\sec x + \tan x| + C$ 229. $\frac{1}{7} \sec^7 x - \frac{1}{5} \sec^5 x + C$ 230. $\frac{1}{8} \sec^7 x \tan x - \frac{3}{16} \sec^5 x \tan x + \frac{1}{64} \sec^3 x \tan x + \frac{3}{128} \sec x \tan x + \frac{3}{128} \ln |\sec x + \tan x| + C$ 231. $\tan x + \frac{2}{3} \tan^3 x + \frac{1}{5} \tan^5 x + C$ 232. $\frac{1}{2} \tan^2 x + \frac{2}{4} \tan^4 x + \frac{1}{6} \tan^6 x + C$ 233. $\frac{1}{3} \tan^3 x + \frac{2}{5} \tan^5 x + \frac{1}{7} \tan^7 x + C$ 234. $\frac{1}{4} \tan^4 x + \frac{2}{6} \tan^6 x + \frac{1}{8} \tan^8 x + C$ 235. $\frac{1}{5} \tan^5 x + \frac{2}{7} \tan^7 x + \frac{1}{9} \tan^9 x + C$
 3a. $x + C$ 3b. $\sin x + C$ 3c. $\frac{1}{2} x + \frac{1}{4} \sin(2x) + C$ 3d. $\sin x - \frac{1}{3} \sin^3 x + C$ 3e. $\frac{1}{4} \cos^3 x \sin x + \frac{3}{8} \cos x \sin x + \frac{3}{8} x + C$ 3f. $-\cos x + C$
 3g. $-\frac{1}{2} \cos^2 x + C = \frac{1}{2} \sin^2 x + C = -\frac{1}{4} \cos(2x) + C$ 3h. $-\frac{1}{3} \cos^3 x + C$ 3i. $-\frac{1}{4} \cos^4 x + C = \frac{1}{2} \sin^2 x - \frac{1}{4} \sin^4 x + C$ 3j. $-\frac{1}{5} \cos^5 x + C$
 3k. $\frac{1}{2} x - \frac{1}{4} \sin(2x) + C$ 3l. $\frac{1}{3} \sin^3 x + C$ 3m. $\frac{1}{8} x - \frac{3}{32} \sin(4x) + C = -\frac{1}{4} \cos^3 x \sin x + \frac{1}{8} \cos x \sin x + \frac{1}{8} x + C$ 3n. $\frac{1}{5} \sin^3 x - \frac{1}{5} \sin^5 x + C$
 3o. $-\frac{1}{6} \cos^5 x \sin x + \frac{1}{24} \cos^3 x \sin x + \frac{1}{16} \cos x \sin x + \frac{1}{16} x + C$ 3p. $\frac{1}{3} \cos^3 x - \cos x + C$ 3q. $\frac{1}{4} \sin^4 x + C = \frac{1}{4} \cos^4 x - \frac{1}{2} \cos^2 x + C$
 3r. $\frac{1}{5} \cos^5 x - \frac{1}{3} \cos^3 x + C$ 3s. $\frac{1}{4} \sin^4 x - \frac{1}{6} \sin^6 x + C = \frac{1}{6} \cos^6 x - \frac{1}{4} \cos^4 x + C$ 3t. $\frac{1}{7} \cos^7 x - \frac{1}{5} \cos^5 x + C$ 3u. $-\frac{1}{4} \sin^3 x \cos x - \frac{3}{8} \sin x \cos x + \frac{3}{8} x + C$ 3v. $\frac{1}{5} \sin^5 x + C$ 3w. $\frac{1}{6} \sin^5 x \cos x - \frac{1}{24} \sin^3 x \cos x - \frac{1}{16} \sin x \cos x + \frac{1}{16} x + C$ 3x. $\frac{1}{5} \sin^5 x - \frac{1}{7} \sin^7 x + C$
 3y. $-\frac{1}{128} \sin^3(2x) \cos(2x) - \frac{3}{256} \sin(2x) \cos(2x) + \frac{3}{128} x + C$ 3z. $-\cos x + \cos^2 x - \frac{1}{5} \cos^3 x + C$ 327. $\frac{1}{6} \sin^6 x + C = -\frac{1}{2} \cos^2 x + \frac{2}{3} \cos^3 x - \frac{1}{4} \cos^4 x + C$ 328. $-\frac{1}{3} \cos^3 x + \frac{1}{2} \cos^4 x - \frac{1}{5} \cos^5 x + C$ 329. $\frac{1}{6} \sin^6 x - \frac{1}{8} \sin^5 x + C = -\frac{1}{4} \cos^4 x + \frac{2}{5} \cos^5 x - \frac{1}{6} \cos^6 x + C$ 330. $-\frac{1}{5} \cos^5 x + \frac{1}{3} \cos^6 x - \frac{1}{7} \cos^7 x + C$ 331. $-\frac{1}{6} \sin^5 x \cos x - \frac{5}{24} \sin^3 x \cos x - \frac{5}{16} \sin x \cos x + \frac{5}{16} x + C$ 332. $\frac{1}{7} \sin^7 x + C$
 333. $\frac{1}{8} \sin^7 x \cos x - \frac{3}{48} \sin^5 x \cos x - \frac{5}{192} \sin^3 x \cos x - \frac{5}{128} \sin x \cos x + \frac{5}{128} x + C$ 334. $\frac{1}{7} \sin^7 x - \frac{1}{9} \sin^9 x + C$ 335. $-\frac{1}{10} \sin^9 x \cos x + \frac{11}{80} \sin^7 x \cos x - \frac{3}{48} \sin^5 x \cos x - \frac{5}{640} \sin^3 x \cos x - \frac{15}{1280} \sin x \cos x + \frac{15}{1280} x + C$