MATH 220–01 (Kunkle), Quiz 9 Name: ______________________________

10 pts, Take-home Due in class April 12, 2017

1a (5 pts). Find the second degree Taylor polynomial $T_2(x)$ centered at $a = 0$ for the function $g(x) = \frac{1}{8}(1-x)^8$.

1b (5 pts). Use Taylor's Theorem to find an upper bound on the absolute error $|g(x) - T_2(x)|$ on the interval $(-1, 1)$. You can leave unfinished arithmetic in your answer.

1a.(Source: 11.11.13-22) We need the first two derivatives of $g(x)$ to find $T_2(x)$, and the third for the error analysis in 1b.

n	$g^{(n)}(x)$	$g^{(n)}(0)/n!$
0	$\frac{1}{8}(1-x)^8$	$\frac{1}{8}$
1	$-(1-x)^7$	-1
2	$7(1-x)^6$	$\frac{7}{2}$
3	$-42(1-x)^5$	(not relevant)

Therefore $T_2(x) = \frac{1}{8} - x + \frac{7}{2}x^2$. **(done)**

You could also have found $T_2(x)$ by finding the quadratic part of the MacLaurin series for $\frac{1}{8}(1-x)^8$:

$$\frac{1}{8}(1-x)^8 = \frac{1}{8}\sum_{n=0}^{\infty}\binom{8}{n}(-1)^n x^n = \frac{1}{8}\left(1 - \frac{8}{1}x + \frac{8\cdot 7}{2\cdot 1}x^2 + \cdots\right).$$

The last step above is the same as expanding $(1-x)^8$ via Pascal's Triangle.

1b. The maximum of $|g'''(x)| = 42|1-x|^5$ on $[-1, 1]$ occurs when $|1-x|$ is greatest, and this is at $x = -1$, so $|g'''(x)| \le 42 \cdot 2^5$. By Taylor's Theorem,

$$|g(x) - T_2(x)| = \frac{|g'''(c)|}{3!}|x|^3 \le \frac{42\cdot 2^5}{6}\cdot 1^3 \ (= 7\cdot 2^5).$$

(done)