MATH 220–01 (Kunkle), Quiz 8 Name: ____________________

10 pts, 10 minutes April 4, 2017

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1 (10 pts). Find power series representations of the following functions:

a. $f(x) = \dfrac{1}{(1-2x)^3}$ b. $g(x) = x\tan^{-1}(x^2)$

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1. Both of these series follow from the geometric series:

(1)
$$(1-x)^{-1} = \sum_{n=0}^{\infty} x^n.$$

Differentiate this twice and obtain

$$2(1-x)^{-3} = \sum_{n=2}^{\infty} n(n-1)x^{n-2}$$

Divide by 2, and replace x with $2x$, and the answer to part a. is

$$(1-2x)^{-3} = \sum_{n=2}^{\infty} n(n-1)2^{n-3}x^{n-2}$$

(Source: 11.9.17)

You can remember the MacLaurin series for $\arctan x$ by repeating the steps we saw in class. In (1), replace x with $-x^2$ and integrate the result:

$$\tan^{-1} x = \int \frac{1}{1+x^2}\,dx = \int \sum_{n=0}^{\infty} (-1)^n x^{2n}\,dx = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$$

The constant of integration is zero, as seen in class. So, the answer to part b. is

$$x\sum_{n=0}^{\infty} (-1)^n \frac{(x^2)^{2n+1}}{2n+1} = x\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+2}}{2n+1} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+3}}{2n+1}$$

(Source: 11.9.16)