MATH 220–01 (Kunkle), Quiz 7
10 pts, 10 minutes

Name: ____________________
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1 (10 pts). Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{n+3}{(n+1)^{3/2}}$$

1.(Source: 11.4.22) Limit-compare the given series with the p-series $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$:

$$\lim_{n\to\infty} \frac{\frac{1}{n^{1/2}}}{\frac{n+3}{(n+1)^{3/2}}} = \lim_{n\to\infty} \frac{(n+1)^{3/2}}{n^{1/2}(n+3)} \lim_{n\to\infty} \frac{(n+1)^{3/2}}{n^{3/2}+3n^{1/2}}$$

Factor out and cancel the dominant term $n^{3/2}$ from the top and bottom:

$$\lim_{n\to\infty} \frac{n^{3/2}(1+\frac{1}{n})^{3/2}}{n^{3/2}(1+\frac{3}{n})} = \lim_{n\to\infty} \frac{(1+\frac{1}{n})^{3/2}}{(1+\frac{3}{n})} = \frac{(1+0)^{3/2}}{1+0} = 1.$$

Since this limit is positive and finite, the Limit Comparison Test says that the given series and $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ either both converge or both diverge. The p-series diverges because $p = 1/2 < 1$, and therefore so does $\sum_{n=1}^{\infty} \frac{n+3}{(n+1)^{3/2}}$. **(done)**

Here's another solution Chandler Deloach and I worked out together.

$$\frac{n+3}{(n+1)^{3/2}} > \frac{n+1}{(n+1)^{3/2}} = \frac{1}{(n+1)^{1/2}} > 0.$$

Notice that

$$\sum_{n=1}^{\infty} \frac{1}{(n+1)^{1/2}} = \frac{1}{2^{1/2}} + \frac{1}{3^{1/2}} + \frac{1}{4^{1/2}} + \cdots = \sum_{m=2}^{\infty} \frac{1}{m^{1/2}}$$

is a divergent p-series beginning at $m = 2$ instead of $m = 1$. Therefore, the larger series $\sum_{n=1}^{\infty} \frac{n+3}{(n+1)^{3/2}}$ diverges by the Comparison Test. **(done)**