
1 (10 pts). Evaluate the improper integral or show that it diverges: $\int_0^{\infty} \frac{x dx}{(x^2 + 5)^{3/2}}$

1.(Source: 7.7.11) Substitute $u = x^2 + 5$, $du = 2x dx$ to evaluate the indefinite integral:

$$\int \frac{x dx}{(x^2 + 5)^{3/2}} = \int \frac{1}{2} u^{-3/2} du = -u^{-1/2} + C = \frac{-1}{\sqrt{x^2 + 5}} + C.$$

Now the improper integral equals

$$\begin{aligned} \int_0^{\infty} \frac{x dx}{(x^2 + 5)^{3/2}} &= \lim_{N \rightarrow \infty} \int_0^N \frac{x dx}{(x^2 + 5)^{3/2}} \\ &= \lim_{N \rightarrow \infty} \left. \frac{-1}{\sqrt{x^2 + 5}} \right|_0^N \\ &= \lim_{N \rightarrow \infty} \left(\frac{-1}{\sqrt{N^2 + 5}} + \frac{1}{\sqrt{5}} \right) \\ &= \frac{1}{\sqrt{5}}. \end{aligned}$$

(done)