1 (10 pts). Integrate: $\int \frac{\sqrt{9-x^2}}{x^2} \, dx$

Solution:

1. (Source: 7.3.4) Use trig substitution. Because we want

$$9 - x^2 = 9 - 9\sin^2 \theta = 9(1 - \sin^2 \theta) = 9 \cos^2 \theta,$$

we let

$$x = 3\sin \theta, \quad dx = 3\cos \theta \, d\theta.$$

Now the integral becomes

$$\int \frac{\sqrt{9\cos^2 \theta}}{9\sin^2 \theta} \cdot 3\cos \theta \, d\theta = \int \frac{\cos^2 \theta}{\sin^2 \theta} \, d\theta = \int \cot^2 \theta \, d\theta = \int (\csc^2 \theta - 1) \, d\theta = -\cot \theta - \theta + C.$$

To rewrite this answer in terms of the original variable $x$, draw a right triangle with interior angle $\theta$. Label two sides using $\sin \theta = x/3$, and then find the third side by the Pythagorean theorem:

So the integral equals $-\cot \theta - \theta + C = -\frac{\sqrt{9-x^2}}{x} - \sin^{-1}(x/3) + C$. 
1 (10 pts). Integrate: \[ \int \frac{\sqrt{9-x^2}}{x^2} \, dx \]

**Solution:**

1. (Source: 7.3.4) Use trig substitution. Because we want

\[ 9 - x^2 = 9 - 9 \sin^2 \theta = 9(1 - \sin^2 \theta) = 9 \cos^2 \theta, \]

we let

\[ x = 3 \sin \theta, \quad dx = 3 \cos \theta \, d\theta. \]

Now the integral becomes

\[ \int \frac{\sqrt{9 \cos^2 \theta}}{9 \sin^2 \theta} 3 \cos \theta \, d\theta = \int \frac{\cos^2 \theta}{\sin^2 \theta} \, d\theta = \int \cot^2 \theta \, d\theta = \int (\csc^2 \theta - 1) \, d\theta = -\cot \theta - \theta + C. \]

To rewrite this answer in terms of the original variable \( x \), draw a right triangle with interior angle \( \theta \). Label two sides using \( \sin \theta = x/3 \), and then find the third side by the Pythagorean theorem:

\[ \sqrt{9-x^2} \]

So the integral equals \( -\cot \theta - \theta + C = -\dfrac{\sqrt{9-x^2}}{x} - \sin^{-1}(x/3) + C. \)
1 (10 pts). Integrate: \( \int \frac{\sqrt{9 - x^2}}{x^2} \, dx \)

**Solution:**

1. (Source: 7.3.4) Use trig substitution. Because we want 

\[
9 - x^2 = 9 - 9 \sin^2 \theta = 9(1 - \sin^2 \theta) = 9 \cos^2 \theta,
\]

we let 

\[
x = 3 \sin \theta, \quad dx = 3 \cos \theta \, d\theta.
\]

Now the integral becomes

\[
\int \frac{\sqrt{9 \cos^2 \theta}}{9 \sin^2 \theta} \, 3 \cos \theta \, d\theta = \int \frac{\cos^2 \theta}{\sin^2 \theta} \, d\theta = \int \cot^2 \theta \, d\theta = \int (\csc^2 \theta - 1) \, d\theta = -\cot \theta - \theta + C
\]

To rewrite this answer in terms of the original variable \( x \), draw a right triangle with interior angle \( \theta \). Label two sides using \( \sin \theta = x/3 \), and then find the third side by the Pythagorean theorem:

![Right triangle](image)

So the integral equals 

\[
-\cot \theta - \theta + C = -\frac{\sqrt{9 - x^2}}{x} - \sin^{-1}(x/3) + C.
\]
1 (10 pts). Integrate: \[ \int \frac{\sqrt{9-x^2}}{x^2} \, dx \]

**Solution:**

1. (Source: 7.3.4) Use trig substitution. Because we want

\[ 9 - x^2 = 9 - 9\sin^2 \theta = 9(1 - \sin^2 \theta) = 9\cos^2 \theta, \]

we let

\[ x = 3\sin \theta, \quad dx = 3\cos \theta \, d\theta. \]

Now the integral becomes

\[
\int \frac{\sqrt{9\cos^2 \theta}}{9\sin^2 \theta} \cdot 3\cos \theta \, d\theta = \int \frac{\cos^2 \theta}{\sin^2 \theta} \, d\theta = \int \cot^2 \theta \, d\theta = \int (\csc^2 \theta - 1) \, d\theta = -\cot \theta - \theta + C
\]

To rewrite this answer in terms of the original variable \( x \), draw a right triangle with interior angle \( \theta \). Label two sides using \( \sin \theta = x/3 \), and then find the third side by the Pythagorean theorem:

\[
\begin{array}{c}
\text{3} \\
\theta \\
x
\end{array} \quad \begin{array}{c}
\text{3} \\
\theta \\
\sqrt{9-x^2}
\end{array}
\]

So the integral equals \(-\cot \theta - \theta + C = -\frac{\sqrt{9-x^2}}{x} - \sin^{-1}(x/3) + C\).