MATH 220–01 (Kunkle), Quiz 2 | Name: ______________________

10 pts, 10 minutes | Jan 24, 2017

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1 (10 pts). Integrate: $\int (\ln x)^2\,dx$

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1. (Source: 7.1.15 8e)

Solution One:

Use integration by parts:

$$u = (\ln x)^2 \quad dv = dx \qquad du = 2(\ln x)\frac{dx}{x} \quad v = x$$

and the indefinite integral becomes

$$\int (\ln x)^2\,dx = \int u\,dv = uv - \int v\,du = x(\ln x)^2 - 2\int (\ln x)\,dx.$$

Use parts again:

$$U = \ln x \quad dV = dx \qquad dU = \frac{dx}{x} \quad V = x$$

and then the integral is

$$\begin{aligned}\int (\ln x)^2\,dx &= x(\ln x)^2 - 2\int (\ln x)\,dx \\ &= x(\ln x)^2 - 2\left[x\ln x - \int dx\right] \\ &= x(\ln x)^2 - 2x\ln x + 2x + C \\ &= x\big((\ln x)^2 - 2\ln x + 2\big) + C.\end{aligned}$$

Solution Two:

First make the substitution $y = \ln x$. Then $dy = \frac{1}{x}\,dx$, so $dx = x\,dy = e^y\,dy$, and the integral becomes

$$\int (\ln x)^2\,dx = \int y^2 e^y\,dy.$$

Now use integration by parts twice as in Example 3, p.474, resulting in

$$e^y(y^2 - 2y + 2) + C = x\big((\ln x)^2 - 2\ln x + 2\big) + C.$$