MATH 203–03 (Kunkle), Quiz 4 Name: ______________________________
10 pts, 10 minutes Feb 17, 2023

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1 (10 pts). Suppose that columns of B are linearly dependent. Explain why the columns of AB must also be linearly dependent.
You should assume that the sizes of A and B allow these matrices to be multiplied.

Solution:
1(10 pts).(Source: 2.1.22) Recall that the columns of a matrix C are linearly dependent if and only if $C\mathbf{x} = \mathbf{0}$ for some nonzero vector $\mathbf{x}$.

Since the columns of B are linearly dependent, there's a nonzero vector $\mathbf{x}$ for which $B\mathbf{x} = \mathbf{0}$. But then $AB\mathbf{x} = A\mathbf{0} = \mathbf{0}$, so the columns of AB are also linearly dependent.

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1 (10 pts). Find the inverse of

$$\begin{bmatrix} 3 & -1 & 0 \\ 0 & 1 & 2 \\ -6 & 2 & 1 \end{bmatrix}$$

or show that it does not exist.

Solution:

1(10 pts).(Source: 2.2.31-32) Augment the given matrix with the identity and row-reduce.

row operation	**result**
(beginning matrix)	$\begin{matrix} 3 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ -6 & 2 & 1 & 0 & 0 & 1 \end{matrix}$
$\mathbf{r}_3 \leftarrow \mathbf{r}_3 + 2\mathbf{r}_1$	$\begin{matrix} 3 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{matrix}$
$\mathbf{r}_2 \leftarrow \mathbf{r}_2 - 2\mathbf{r}_3$	$\begin{matrix} 3 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -4 & 1 & -2 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{matrix}$
$\mathbf{r}_1 \leftarrow \mathbf{r}_1 + \mathbf{r}_2$	$\begin{matrix} 3 & 0 & 0 & -3 & 1 & -2 \\ 0 & 1 & 0 & -4 & 1 & -2 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{matrix}$
$\mathbf{r}_1 \leftarrow \frac{1}{3}\mathbf{r}_1$	$\begin{matrix} 1 & 0 & 0 & -1 & \frac{1}{3} & -\frac{2}{3} \\ 0 & 1 & 0 & -4 & 1 & -2 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{matrix}$

Therefore the inverse matrix is

$$\begin{bmatrix} -1 & \frac{1}{3} & -\frac{2}{3} \\ -4 & 1 & -2 \\ 2 & 0 & 1 \end{bmatrix}$$