MATH 203-01 QUIZ 7 OCT 17, 2014

Find a basis and the dimension of the following, if

$$A = \begin{pmatrix} 1 & 0 & 4 & 5 \\ -1 & 0 & 2 & 4 \end{pmatrix}$$

1. Col A 2. Row A 3. Nul A

Solution: Row reduce

$$\begin{pmatrix} 1 & 0 & 4 & 5 \\ -1 & 0 & 2 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 4 & 5 \\ 0 & 0 & 6 & 9 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 4 & 5 \\ 0 & 0 & 1 & 3/2 \end{pmatrix}$$

$$\sim \begin{pmatrix} \boxed{1} & 0 & 0 & -1 \\ 0 & 0 & \boxed{1} & 3/2 \end{pmatrix} = EF(A)$$

$\boxed{}$ = pivots

1. Basis Col A = {pivot cols A} = $\left\{ \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 4 \\ 2 \end{pmatrix} \right\}$. dimension = 2

2. Basis Row A = {pivot rows of EF(A)}

$$= \left\{ \begin{matrix} (1 & 0 & 0 & -1) \\ (0 & 0 & 1 & 3/2) \end{matrix} \right\}$$. dimension = 2

(In this example, rows of A are also a basis for Row A since they are a linearly independent spanning set for Row A.)

3. $\text{Nul } A = \{ x \mid Ax = 0 \} = \left\{ \begin{pmatrix} x_4 \\ x_2 \\ -3/2 x_4 \\ x_4 \end{pmatrix} : x_2, x_4 \in \mathbb{R} \right\}$

$$= \text{span} \left\{ \begin{pmatrix} 1 \\ 0 \\ -3/2 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \right\}$$

Basis for Nul A = $\left\{ \begin{pmatrix} 1 \\ 0 \\ -3/2 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \right\}$. Dimension = 2.