In the statements below, A and B are matrices, V is a vector space, $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are vectors in V, and T is a linear map from V into another vector space.

Identify each statement as True or False.

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1 (1 pts). $\operatorname{Nul} A$ is the set of all solutions $\mathbf{x}$ to $A\mathbf{x} = \mathbf{0}$.

2 (1 pts). $\mathbb{R}^5$ is a subspace of $\mathbb{R}^6$.

3 (1 pts). If $A \sim B$, then the pivot columns of B are a basis for $\operatorname{Col} A$.

4 (1 pts). $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$ is a basis for the column space of $\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$.

5 (1 pts). If the set of vectors $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly dependent, then $\mathbf{u}$ must be a linear combination of $\mathbf{v}$ and $\mathbf{w}$.

6 (1 pts). In any vector space, $\{\mathbf{0}\}$ is linearly independent.

7 (1 pts). The set $\{T(\mathbf{x}) \mid \mathbf{x} \in V\}$ is called the domain of T.

8 (1 pts). If $T(\mathbf{u}) = T(\mathbf{v})$, then $\mathbf{u} - \mathbf{v}$ is in the kernel of T.

9 (1 pts). $\operatorname{span}\{1 - t, 2 + t, t^2 - 1\}$ is a subspace of $\mathbb{P}_3$.

10 (1 pts). The three vectors $\left\{ \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \right\}$ are linearly independent.

Solutions:

1. **T**. Definition, p. 226.

2. **F**. $\mathbb{R}^5$ and $\mathbb{R}^6$ are **disjoint**; that is, they have no vectors in common.

3. **F**. The pivot columns of B are a basis for $\operatorname{Col} B$, but this in not necessarily equal $\operatorname{Col} A$. For example, let $A = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

4. **T**. Since the matrix has a pivot in each row and column, its column space is $\mathbb{R}^3$, and the given set is the standard basis for $\mathbb{R}^3$.

5. **F**. One of the three must be a linear combination of the other two, but it needn't be $\mathbf{u}$. For example, consider $\{1, t, 2t\}$ in $\mathbb{P}_1$.

6. **F**. $\{\mathbf{0}\}$ is linearly dependent, since $1 \cdot \mathbf{0} = \mathbf{0}$ (without 1 equaling 0).

7. **F**. The set in question is the range of T, not the domain.

8. **T**. kernel $T = \{\mathbf{x} \in V \mid T(\mathbf{x}) = \mathbf{0}\}$. By linearity, $T(\mathbf{u} - \mathbf{v}) = T(\mathbf{u}) - T(\mathbf{v}) = \mathbf{0}$.

9. **T**. The span of any set of vectors is a vector space, and the three polynomials all belong to $\mathbb{P}_3$, the polynomials of degree 3 or less.

10. **T**. $a \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} + c \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} a & c \\ a & b \end{pmatrix}$, and if this equals $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, then $a = b = c = 0$.