In the statements below, $A$ and $B$ are matrices, $V$ is a vector space, $u$, $v$, and $w$ are vectors in $V$, and $T$ is a linear map from $V$ into another vector space.

Identify each statement as True or False.

1 (1 pts). $\text{Nul } A$ is the set of all solutions $x$ to $Ax = 0$.
2 (1 pts). $\mathbb{R}^5$ is a subspace of $\mathbb{R}^6$.
3 (1 pts). If $A \sim B$, then the pivot columns of $B$ are a basis for $\text{Col } A$.
4 (1 pts). \[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\] is a basis for the column space of \[
\begin{bmatrix}
1 & -1 & 0 \\
0 & 1 & 1 \\
0 & 0 & 1
\end{bmatrix}
\].
5 (1 pts). If the set of vectors $\{u, v, w\}$ is linearly dependent, then $u$ must be a linear combination of $v$ and $w$.
6 (1 pts). In any vector space, $\{0\}$ is linearly independent.
7 (1 pts). The set $\{T(x) \mid x \in V\}$ is called the domain of $T$.
8 (1 pts). If $T(u) = T(v)$, then $u - v$ is in the kernel of $T$.
9 (1 pts). $\text{span}\{1-t, 2+t, t^2-1\}$ is a subspace of $\mathbb{P}_3$.
10 (1 pts). The three vectors \[
\begin{bmatrix}
1 & 0 \\
1 & 0
\end{bmatrix}, \begin{bmatrix}
0 & 0 \\
0 & 1
\end{bmatrix}, \begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix}
\] are linearly independent.

Solutions:

1. **T.** Definition, p. 226.
2. **F.** $\mathbb{R}^5$ and $\mathbb{R}^6$ are disjoint; that is, they have no vectors in common.
3. **F.** The pivot columns of $B$ are a basis for $\text{Col } B$, but this in not necessarily equal $\text{Col } A$.
   For example, let $A = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.
4. **T.** Since the matrix has a pivot in each row and column, its column space is $\mathbb{R}^3$, and
   the given set is the standard basis for $\mathbb{R}^3$.
5. **F.** One of the three must be a linear combination of the other two, but it needn’t be $u$.
   For example, consider $\{1, t, 2t\}$ in $\mathbb{P}_1$.
6. **F.** $\{0\}$ is linearly dependent, since $1 \cdot 0 = 0$ (without 1 equaling 0).
7. **F.** The set in question is the range of $T$, not the domain.
8. **T.** $\text{kernel } T = \{x \in V \mid T(x) = 0\}$. By linearity, $T(u - v) = T(u) - T(v) = 0$.
9. **T.** The span of any set of vectors is a vector space, and the three polynomials all belong
to $\mathbb{P}_3$, the polynomials of degree 3 or less.
10. **T.** $a \begin{bmatrix} 1 & 0 \\
1 & 0
\end{bmatrix} + b \begin{bmatrix} 0 & 0 \\
0 & 1
\end{bmatrix} + c \begin{bmatrix} 0 & 1 \\
0 & 0
\end{bmatrix} = \begin{bmatrix} a & c \\
\end{bmatrix}$, and if this equals
    $\begin{bmatrix} 0 & 0 \\
0 & 0
\end{bmatrix}$, then $a = b = c = 0$. 