MATH 203-01 Quiz 4 Sept 26, 2014

Find the determinant: $\begin{vmatrix} 4 & 2 & 1 & 0 \\ 4 & -1 & 3 & -1 \\ -1 & 6 & 8 & 0 \\ 1 & 1 & -1 & 0 \end{vmatrix}$

There are many correct solutions. I started by expanding along column 4, and then proceeded with row reduction.

$$= -1 \cdot \begin{vmatrix} 4 & 2 & 1 \\ -1 & 6 & 8 \\ 1 & 1 & -1 \end{vmatrix} \overset{1}{=} \begin{vmatrix} 1 & 1 & -1 \\ -1 & 6 & 8 \\ 4 & 2 & 1 \end{vmatrix}$$

$$\overset{2}{=} \begin{vmatrix} 1 & 1 & -1 \\ 0 & 7 & 7 \\ 0 & -2 & 5 \end{vmatrix} \overset{3}{=} 7 \begin{vmatrix} 1 & 1 & -1 \\ 0 & 1 & 1 \\ 0 & -2 & 5 \end{vmatrix}$$

$$\overset{4}{=} 7 \begin{vmatrix} 1 & 1 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 7 \end{vmatrix} \overset{5}{=} 7 \cdot 1 \cdot 1 \cdot 7 = 49.$$

Notes. 1. $r_3 \leftrightarrow r_1$ 2. $r_2 \leftarrow r_2 + r_1$, and $r_3 \leftarrow r_3 - 4r_1$
3. Factor 7 from r_2 4. $r_3 \leftarrow r_3 + 2r_2$
5. The determinant of a triangular matrix is the product of its diagonal elements.