MATH 203-01 QUIZ 10 NOV 7, 2014

Find the eigenvalues and bases for the associated eigenspaces of $A = \begin{pmatrix} 3 & 5 \\ -1 & 1 \end{pmatrix}$.

Solution: Find A's eigenvalues:

$$|A-\lambda I| = \begin{vmatrix} 3-\lambda & 5 \\ -1 & 1-\lambda \end{vmatrix} = (3-\lambda)(1-\lambda) - (-5) = \lambda^2 - 4\lambda + 8$$
$$= \lambda^2 - 4\lambda + 4 + 4$$
$$= (\lambda-2)^2 + 2^2.$$

eigenvalues are $\lambda = 2 \pm 2i$.

Next find a basis for each eigenspace $= \mathrm{Nul}(A-\lambda I)$.

$\lambda = 2-2i$. $A-\lambda I = \begin{pmatrix} 1+2i & 5 \\ -1 & -1+2i \end{pmatrix} \sim \begin{pmatrix} 1 & 1-2i \\ 1+2i & 5 \end{pmatrix} \overset{*}{\sim} \begin{pmatrix} 1 & 1-2i \\ 0 & 0 \end{pmatrix}$

(* either observe $(1+2i)(1-2i) = 5$, or trust that your work to here is correct so $A-\lambda I$ must have a free variable.)

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} (-1+2i)x_2 \\ x_2 \end{pmatrix} = x_2 \begin{pmatrix} -1+2i \\ 1 \end{pmatrix}. \quad \text{Basis} = \left\{ \begin{pmatrix} -1+2i \\ 1 \end{pmatrix} \right\}$$

The eigenvectors of $\lambda = 2+2i$ are conjugates those of $\lambda = 2-2i$.

$\text{Basis} = \left\{ \begin{pmatrix} -1-2i \\ 1 \end{pmatrix} \right\}$. done

---

BTW, this means $A = PCP^{-1}$ where $P = \begin{pmatrix} -1 & 2 \\ 1 & 0 \end{pmatrix}$ & $C = \begin{pmatrix} 2 & -2 \\ 2 & 2 \end{pmatrix}$.

To obtain this P and C, I used $\lambda = 2-2i = a-bi$, so that $C = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ and columns of P are the real and imaginary parts of the eigenvector $\begin{pmatrix} -1+2i \\ 1 \end{pmatrix}$.