

1. Find the general solution to $AX = b$, where

$$A = \begin{pmatrix} 1 & 0 & -2 & 2 \\ 1 & 1 & 0 & 2 \\ 2 & 1 & -2 & 5 \end{pmatrix} \text{ and } b = \begin{pmatrix} 7 \\ 3 \\ 15 \end{pmatrix}.$$

2. Based on your work in 1., would you say that the columns of A span \mathbb{R}^4 , \mathbb{R}^3 , or neither? Why?

Solutions: 1. $(A \vdots b) =$

$$\begin{pmatrix} 1 & 0 & -2 & 2 & 7 \\ 1 & 1 & 0 & 2 & 3 \\ 2 & 1 & -2 & 5 & 15 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 0 & -2 & 2 & 7 \\ 0 & 1 & 2 & 0 & -4 \\ 2 & 1 & -2 & 5 & 15 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 0 & -2 & 2 & 7 \\ 0 & 1 & 2 & 0 & -4 \\ 0 & 1 & 2 & 1 & 1 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 0 & -2 & 2 & 7 \\ 0 & 1 & 2 & 0 & -4 \\ 0 & 0 & 0 & 1 & 5 \end{pmatrix}$$

end forward phase.

$$\sim \begin{pmatrix} \boxed{1} & 0 & -2 & 0 & -3 \\ 0 & \boxed{1} & 2 & 0 & -4 \\ 0 & 0 & 0 & \boxed{1} & 5 \end{pmatrix}$$

end backward phase.

$\square =$ pivots.

$x_1, x_2, x_4 =$ basic vars.
 $x_3 =$ free var.

general solution:

$$x_1 = -3 + 2x_3$$

$$x_2 = -4 - 2x_3$$

$$x_3 = \text{free}$$

$$x_4 = 5.$$

done 1.

2. Columns of A are in \mathbb{R}^3 , so they can't span \mathbb{R}^4 .
 A has a pivot in every row, so the
 columns of A span \mathbb{R}^3 .

done 2.