No Calcuators

1. Find the derivative of each of the following functions. Do not simplify your answers.

a. $y = 4x^7 - \dfrac{4}{\sqrt[3]{x^2}} + \cot x - e^2 - \cos^{-1} x$

b. $(\sin x)(\cos x) + \ln(\tan x)$

c. $7x^2 - \dfrac{1}{8(\sqrt[3]{x})^4} - 2\tan^{-1} x$

d. $\dfrac{\sec x - \tan x}{\cos \pi + \cot x}$

e. $\ln(x^2 e^x)$

f. $(\csc^2 x)\sqrt{x^{10} + x}$

g. $\sqrt[3]{x \ln|\cos e^x|}$

h. $(2x+3)^{5x}$

i. $(\csc^3 x + \sec^3 x)^4$

j. $(x+1)^{\sqrt{x}}$

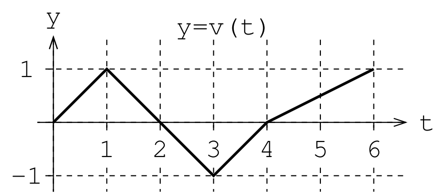
2. Find $\dfrac{dy}{dx}$ along the curve $(x^2+y^2)^5 = x^3 \sin^{-1}(x-y)$.

3. Find $\dfrac{dy}{dx}$ along the curve $3e^{xy} = y - y^2x + 8$.

4. Find an equation for the line tangent to the curve $y = \dfrac{5x}{2x+1}$ at $x = 1$.

5. Find all critical points of the function $\xi(x) = 1 + \cos^2 x - \sin x$ in the interval $[0, 2\pi]$.

6. The graph at the right shows the velocity function $v(t)$ for an object moving along a horizontal axis. (v is measured in m/sec and time t is measured in sec.)



a. Over what intervals of time, if any, is the object moving in the positive direction?

b. Find the object's acceleration at time $t = 2$. Don't forget the units.

c. At what times, if any, does the object's position have a local minimum? At what times, if any, does it have a local maximum?

d. If the object is at position 14 meters at $t = 0$, find its position at time $t = 6$.

e. Find the total distance traveled by the object between times $t = 0$ and $t = 6$.

7. Let $g(x) = x^4 - 4x^3 + 6$.

a. Find the interval(s) on which $g(x)$ is increasing.

b. Find the intervals on which the graph of $g(x)$ is concave up.

c. Find the x-coordinates of all local maxima, local minima, and points of inflection.

d. Find the absolute maximum and minimum values of $g(x)$ on the interval $[2, 4]$

8. Sketch a graph of a function $f(x)$ with all of the following properties.

a. The domain of $f(x)$ is $(-\infty, -3) \cup (-3, 6]$.

b. $f(1) = -3$ and $f(4) = 2$.

c. $\lim_{x\to -3^-} f(x) = -\infty$.

d. $\lim_{x\to -3^+} f(x) = \infty$.

e. $f'(x) < 0$ if $x < -3$ or if $-3 < x < 1$.

f. $f'(x) > 0$ if $1 < x < 6$.

g. $f''(x) < 0$ if $x < -3$ or if $4 < x < 6$.

h. $f''(x) > 0$ if $-3 < x < 4$.

9. Find a number c that satisfies the conclusion of the Mean Value Theorem for the function $w(x) = e^{3x}$ on the interval $[0, 2]$.

10. Evaluate the limits. Write a real number, ∞, $-\infty$, or DNE. Include supporting work.

a. $\displaystyle\lim_{x\to 4^+} \frac{6}{4-x}$

b. $\displaystyle\lim_{x\to -\infty} \frac{\sqrt{3x^2+2}}{2x-3}$

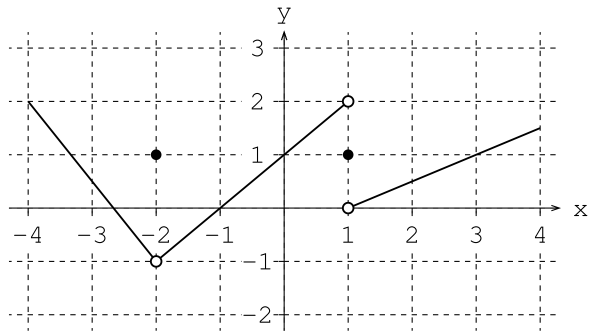
c. $\displaystyle\lim_{x\to 0} (\cos x)^{1/x^2}$

11. Use the graph of $y = f(x)$ at right to find the following. Write a real number, ∞, $-\infty$, or DNE. Supporting work is not required on this problem.

a. $\lim_{x\to 1} f(x)$

b. $\lim_{x\to -2} f(x)$

c. $\lim_{h\to 0} \dfrac{f(3+h)-f(3)}{h}$



12. Can a function be continuous on the interval $[-1,1]$, have an absolute maximum on $[-1,1]$, but have no critical numbers in $[-1,1]$? Give a graph or formula for such a function, or explain why one cannot exist.

13a. Find the linearization $L(x)$ of $f(x) = \sqrt[3]{1+x}$ at $a = 7$.

13b. Use your answer to Problem 13part a to approximate $\sqrt[3]{9}$. Write your answer as a rational number.

14 . Evaluate the following indefinite integrals:

a. $\displaystyle\int \left(\frac{6}{x} - \frac{3}{\sqrt{x}} + e^x\right) dx$

b. $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\, dx$

c. $\displaystyle\int (3 - \sec\theta)\cos\theta\, d\theta$

d. $\displaystyle\int e^{3x}\sqrt{e^{3x}+7}\, dx$

e. $\displaystyle\int \left(x^2 - 3\csc x\cot x - \frac{1}{2x}\right) dx$

f. $\displaystyle\int (3\sin x + \sec^2 x + \pi)\, dx$

g. $\displaystyle\int x\sqrt[4]{x^2+5}\, dx$

h. $\displaystyle\int x\tan x^2 \sec^2 x^2\, dx$

15. Evaluate the definite integrals:

a. $\displaystyle\int_{3\pi/4}^{\pi} \sec^2 x\, dx$

b $\displaystyle\int_1^3 \left(x^2 - 7x - \frac{1}{2x}\right) dx$

c. $\displaystyle\int_1^5 \frac{x}{x^2+1}\, dx$

d. $\displaystyle\int_{1/2}^{1} \frac{\sin^{-1} x}{\sqrt{1-x^2}}\, dx$

16. A particle moves in a straight line. Its velocity at time t is $v(t) = 8\sin t$. If its initial position is $s(0) = 3$, determine its position $s(t)$ as a function of t.

17a. Approximate the value of the definite integral $\int_3^5 \ln x\, dx$ with a Riemann sum using 4 equal subintervals and their right-hand endpoints. You are not required to evaluate your answer.

17b. Is your approximation in Problem 17part a an overestimate or an underestimate of the integral? Explain with a graph.

18 Find all horizontal and vertical asymptotes of $y = \dfrac{x^2 - 4x + 4}{(x-2)(2x+3)}$

19. Write the expression $\int_1^3 p(t)\, dt + \int_3^6 p(t)\, dt - \int_2^6 p(t)\, dt$ as a single definite integral.

20. Find $\int_{-1}^{1} \big(3f(x) - 2g(x) + 5\big)\, dx$ if

$\int_{-1}^{3} f(x)\, dx = 9$, $\int_1^3 f(x)\, dx = 5$, $\int_0^{-1} g(x)\, dx = 3$, and $\int_0^1 g(x)\, dx = 6$.

21 Let $F(x) = \displaystyle\int_{\sqrt{3}}^{6x} (t^2 - 3)\, dt$.

a. Is $F(0)$ positive or negative? Briefly explain.

b. Find $F'(x)$.

22. A 13-foot ladder is leaning against a vertical wall on level ground. If the foot of the ladder is pulled along the ground away from the wall at a speed of 2 feet per second, how fast is top of the ladder sliding down the wall when the top is 5 feet from the ground?

23. A farmer wants to fence an area of 120 square feet in a rectangular field and then divide the field into equal thirds with two fences parallel to one of the sides of the rectangle. Find the shortest length of fencing needed to do this.

24. An object moving along a straight track is at position $s(t)$ at time t. The graph of $s(t)$ appears in the figure. Let v_A denote the object's velocity at the point A; let v_B denote the object's velocity at the point B, etc.. Fill in the blanks with v_A, v_B, v_C, v_D, v_E to make a true statement. (Supporting work not required.)

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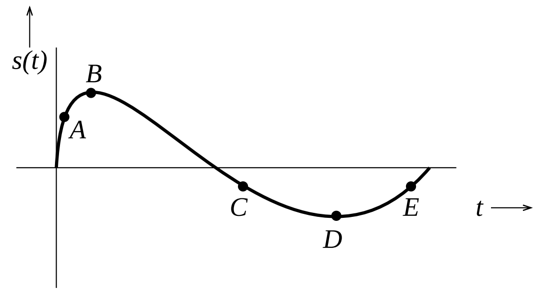


25. State the definition of $\lim_{x\to b} g(x) = M$.

26. State the definition of $\lim_{x\to w} h(x) = -\infty$.

27. Write an ε-δ proof of $\lim_{x\to -2}(-3x+1) = 7$.

28a. Use the limit definition of derivative to find $s'(x)$ if $s(x) = \dfrac{1}{\sqrt{5-x}}$. (You can use the "shortcut" methods of finding derivatives to check your work, but to receive credit on this problem, you must find $f'(x)$ using one of the two limit-definitions of the derivative. You may not use L'Hospital's Rule to help you take the limit.)

28b. Use your answer to 28a (above) to find the slope of the curve $y = s(x)$ at $x = 1$. (You do not have to get 28a correct to receive full credit on 28b.)

28c. Use your answer to 28b (above) to find the equation of the line tangent to $y = s(x)$ at $x = 1$. (You do not have to get 28a correct to receive full credit on 28c.)

29. Repeat Problem 28 with $s(x) = \dfrac{x}{2x-1}$.

30. Repeat Problem 28 with $s(x) = x^4 - 3x^2$.

31. Take the limit, or explain why it does not exist. Complete solutions required.

a. $\displaystyle\lim_{x\to 4} \frac{x^2+x-20}{x^2-16}$

b. $\displaystyle\lim_{x\to -2} \frac{x^2}{(x+2)^2}$

c. $\displaystyle\lim_{x\to 0} x^2\cos\left(\frac{1}{x^2}\right)$

d. $\displaystyle\lim_{x\to \infty} \frac{x^7-9x^{10}}{4x^{10}-3x^2+1}$

e. $\displaystyle\lim_{x\to 0} \left(\frac{1}{x(x+1)} - \frac{1}{x}\right)$

f. $\displaystyle\lim_{x\to \infty} \left(\sqrt{4x^2+3} - 2x\right)$

32. Sketch the graph of a function $g(x)$ that **clearly** satisfies all of the following:

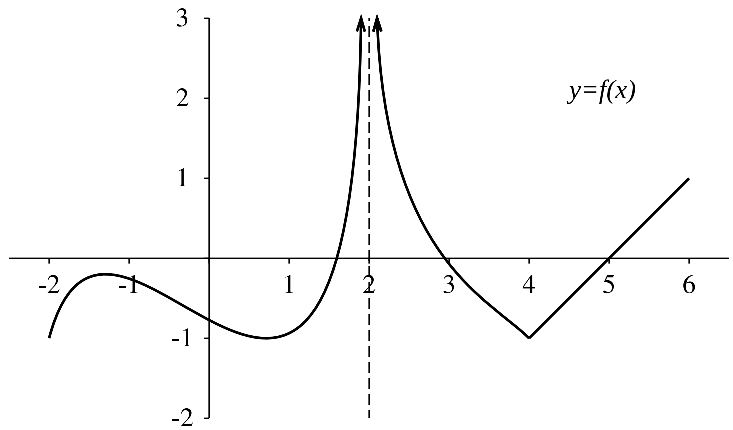
- $\lim_{x\to\infty} g(x) = -2$
- $\lim_{x\to -2^-} g(x) = -\infty$
- $\lim_{x\to-\infty} g(x) = \infty$
- $\lim_{x\to -2^+} g(x) = \infty$
- g is discontinuous at $x = 1$
- g is continuous from the right at $x = 1$

33. Refer to the figure for the graph of $y = f(x)$.

a. Find all x-values in the interval $(-2, 6)$ for which $f'(x)$ does not exist.

b. Find $f'(5)$.

c. Sketch the graph of $y = f'(x)$ on a separate set of axes.



34. Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ if $y = \dfrac{3x+4}{2x+1}$. You are not required to simplify your answers.

35. At what x-values does the graph of $y = x + \cos x$ have a horizontal tangent?

36. Given the following table of values:

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-1	3	4	2	-2
2	-1	-5	3	9
3	2	10	-1	11

a. Find $\dfrac{d}{dx}\left[g\big(f(x)\big)\right]$ at $x = -1$.

b. Find $\dfrac{d}{dx}\big(g(x)\big)^{-1}$ at $x = 2$.

37. An object moving along a number line is at position $s = t^2 + t$ (meters) at time t (seconds).

a. Express the object's velocity and acceleration as functions of t.

b. At what time(s) is the object's velocity equal 0?

c. On what interval(s) of time is the object moving forward?

d. Find the total distance travelled by the object between time $t = -1$ and $t = 1$.

38. State Rolle's Theorem.

39. State the Mean Value Theorem.

40. Show that $p(x) = x^7 + 2x^5 + 7x - 3 = 0$ at exactly one x in the interval $[0, 1]$.

41a. Find the linearization $L(x)$ of the function $g(x) = \ln x$ at $a = 1$.

41b. Make a rough sketch of the graphs of $g(x)$ and $L(x)$ together on the same axes. Label the point of tangency with its coordinates.

41c. If x is near a, is $L(x)$ an underestimate or an overestimate of $g(x)$?

42. Let $f(x) = \ln(x^2 + 4)$. (Note that $f(x)$ is defined on the entire real number line.)

a. On what interval(s) is $f(x)$ increasing?

b. At what x-value(s), if any, does $f(x)$ have a local maximum?

c. At what x-value(s), if any, does $f(x)$ have a local minimum?

d. On what interval(s) is $f(x)$ concave down ?

e. At what x-value(s), if any, does $f(x)$ have an inflection point?

43. Find the absolute maximum and absolute minimum of $\rho(x) = \dfrac{x^2 - 16}{x - 5}$ on the interval $[-5, 4]$.

44. Sketch the graph of a function $\beta(x)$ with all of the following properties.

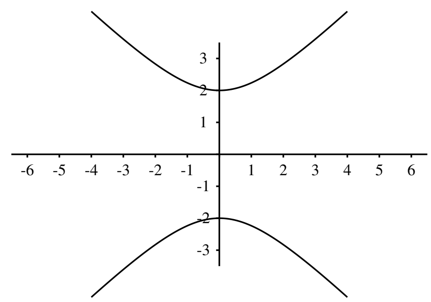
- $\beta(0) = 1$
- $\beta'(-2) = \beta'(0) = 0$
- $\beta'(x) < 0$ on the interval $(-\infty, -2)$
- $\beta'(x) \geq 0$ on the interval $(-2, \infty)$
- $\beta''(x) < 0$ on the interval $(-1, 0)$
- $\beta''(x) > 0$ on the intervals $(-\infty, -1)$ and $(0, \infty)$

45. A weather balloon rising straight up from a level field is tracked by an observer 1 kilometer from the lift-off point. At the moment when the balloon is 3 kilometers from the observer, the angle of elevation of the balloon (from the point of view of the observer) is increasing $\frac{1}{100}$ radians per second. How fast is the balloon rising at that moment? Answer in a complete sentence, including the appropriate units.

46a. At noon, ship A is exactly 1km west of ship B. If B travels east at 2 km/hr and A travels north at 3 km/hr, how fast are they separating at 4pm?

46b. Suppose, instead that the second ship travels northeast along a path that makes a $\pi/3$ angle with the eastward path of the first ship. How fast is the distance between the ships increasing at 1pm?

47. If you walk along the upper branch of the hyperbola $y^2 = x^2 + 4$ (see figure at right) what's the closest you'll come to the point $(2, 0)$?



48. The top and bottom margins of a poster are each 2 inches, and the side margins are each 1 inch. If the area of printed material on the poster is to be 8 square inches, find the smallest possible total surface area of the poster.

49. Find $f(x)$ if $f''(x) = -x^{-2}$ and $f'(-1) = 2$ and $f(-1) = 0$.

50. A marble is thrown downward from a 170 foot tower with initial speed of 4 ft/sec. Assuming that the downward acceleration of the marble is -32 ft/sec^2, express the height of the marble as a function of time.

51 a. Using the table of values below, approximate $\int_0^{30} p(x)\,dx$ with a Riemann Sum using three subintervals and their midpoints.

x	0	5	10	15	20	25	30
$p(x)$	4	6	3	-2	-3	-1	1

You can leave unfinished arithmetic in your answer. For example, if the correct answer were "$\frac{2}{7}(3 \cdot 5^2 - 1)$," then that response would receive full credit.

51b. Repeat, using 6 subintervals and their right endpoints.

52. Find the limits:

a. $\displaystyle\lim_{x\to 0} \frac{\tan x}{\sqrt{x}}$ b. $\displaystyle\lim_{x\to 0} \frac{\sin x}{e^x}$ c. $\displaystyle\lim_{x\to\infty} x \sin\left(\frac{1}{x}\right)$

d. $\displaystyle\lim_{x\to\infty} x - \sqrt{x^2 + 2x}$ e. $\displaystyle\lim_{x\to 0} (1 + 5x)^{1/x}$ f. $\displaystyle\lim_{x\to\infty} x^{\frac{1}{2+\ln x}}$

1. a: $28x^6 + \frac{8}{3}x^{-5/3} - \csc^2 x + (1-x^2)^{-1/2}$. b: $\cos^2 x - \sin^2 x + \sec^2 x \cot x$. c: $14x + \frac{1}{6}x^{-7/3} - 2(1+x^2)^{-1}$ d: $\Big((\sec x \tan x - \sec^2 x)(\cot x - 1) + (\sec x - \tan x)\csc^2 x\Big)(\cot x - 1)^{-2}$ e: $1 + 2/x$ f: $-2\csc^2 x \cot x (x^{10}+x)^{1/2} + \frac{1}{2}\csc^2 x (x^{10}+x)^{-1/2}(10x^9+1)$ g: $\frac{1}{3}(x \ln|\cos e^x|)^{-2/3}(\ln|\cos e^x| - xe^x \tan e^x)$ h: $(2x+3)^{5x}\Big(5\ln(2x+3) + 10x(2x+3)^{-1}\Big)$ i: $4(\csc^3 x + \sec^3 x)^3(-3\csc^3 x \cot x + 3\sec^3 x \tan x)$ j: $(x+1)^{\sqrt{x}}\Big(\frac{1}{2}x^{-1/2}\ln(x+1) + \sqrt{x}(x+1)^{-1}\Big)$. **2**. $\frac{10x(x^2+y^2)^4 - 3x^2 \sin^{-1}(x-y) - x^3(1-(x-y)^2)^{-1/2}}{-10y(x^2+y^2)^4 - x^3(1-(x-y)^2)^{-1/2}}$ **3**. $\frac{3ye^{xy}}{1-2xy-3xe^{xy}}$ **4**. $y - 5/3 = (5/9)(x-1)$ **5**. $\pi/2,\ 3\pi/2,\ 7\pi/6,\ 11\pi/6$ **6**. a:$[0,2] \cup [4,6]$ b: -1 m/sec^2 c: local max at $t=2$; local min at $t=4$ d: 15 e: 3 **7**. a: $[3,\infty)$ b: $(-\infty,0)$ and $(2,\infty)$ c: local min at $x=3$. No local max. Inflect. pts at $x=0$ and $x=2$ d: min = -21, max = 6 **9**. $c = \frac{1}{3}(\ln \frac{1}{6}(e^6-1))$ **10**. a: $-\infty$ b: $-\sqrt{3}/2$ c: $e^{-1/2}$ **11**. a: DNE b: -1 c: 1/2 **12**. Yes. Abs max and min must occur at the endpoints. $f(x) = x$ will work. **13**. a: $2 + \frac{1}{12}(x-7)$ b: $f(8) \approx L(8) = 2 + \frac{1}{12}$ **14**. a: $6\ln|x| - 6\sqrt{x} + e^x + C$ b: $\sin^{-1} x + C$ c: $3\sin\theta - \theta + C$ d: $\frac{2}{9}(e^{3x}+7)^{3/2} + C$ e: $x^3/3 + 3\csc x - \frac{1}{2}\ln|x| + C$ f: $-3\cos x + \tan x + \pi x + C$ g: $\frac{2}{5}(x^2+5)^{5/4} + C$ h: $\frac{1}{4}\tan^2 x^2 + C$ **15**. a: 1 b: $\frac{-58}{3} - \frac{1}{2}\ln 3$ c: $\frac{1}{2}\ln 13$ d: $\frac{1}{9}\pi^2$ **16**. $11 - 8\cos t$ **17**. a: $I \approx \frac{1}{2}(\ln 3.5 + \ln 4 + \ln 4.5 + \ln 5)$ b: $\ln x$ increases, so answer to a is an Overestimate. **18**. HA is $y = \frac{1}{2}$. VA is $x = \frac{-3}{2}$. **19**. $\int_1^2 p(t)\,dt$ **20**. 16 **21**. a: positive, b.c. integrand is negative and limits are in reverse order. b: $6(36x^2 - 3)$ **22**. 24/5 ft/sec **23**. $4 \cdot 60^{1/2} + 240 \cdot 60^{-1/2}$ **24**. two correct answers: $v_C \le v_B \le v_D \le v_E \le v_A$, or $v_C \le v_D \le v_B \le v_E \le v_A$ **25**. For every $\varepsilon > 0$, there exists a $\delta > 0$ so that $0 < |x-b| < \delta$ implies $|g(x) - M| < \varepsilon$. **26**. For every real number M, there exists a $\delta > 0$ so that $0 < |x-w| < \delta$ implies $h(x) < M$. **27**. Assume $\varepsilon > 0$. Choose $\delta = \varepsilon/3$. Then $0 < |x+2| < \delta$ implies that $\varepsilon > 3|x+2| = |3x+6| = |(-3x+1) - 7|$, as desired. **28**. a: $s'(x) = \frac{1/2}{(5-x)^{3/2}}$. b: 1/16. c: $y - \frac{1}{2} = \frac{1}{16}(x-1)$. **29**. a: $s'(x) = -(2x-1)^{-2}$. b: -1. c: $y - 1 = -1(x-1)$. **30**. a: $s'(x) = 4x^3 - 6x$. b: -2. c: $y + 2 = -2(x-1)$. **31**. a: 9/8 b: ∞ c: 0 d: $-9/4$ e: -1 f: 0 **33**. a: $x = 2, x = 4$ b: 1 **34**. $y' = \frac{-5}{(2x+1)^2}$. $y'' = \frac{20}{(2x+1)^3}$. **35**. $x = \frac{\pi}{2} + 2n\pi$, where n = any integer. **36**. a: 44. b: -1 **37**. a: $v = 2t+1$, $a = 2$. b: $t = -1/2$. c: $(-1/2, \infty)$. d: 5/2. **38**. See text. **39**. See text. **40**. $p(0) = -3 < 0 < 7 = p(1)$. Since p is a polynomial and continuous and differentiable everywhere, Intermediate Value Theorem $\Longrightarrow p(c) = 0$ for at least one number c in $(0,1)$. Rolle's $\Longrightarrow$ that between any two zeros of $p(x)$ there must be a zero of $p'(x)$. However, $p'(x) = 7x^6 + 10x^4 + 7 \ge 7$ for all real x, so there cannot be more than one such number c. **41**. a: $x - 1$. c: $L(x) \ge \ln x$ (because $\ln x$ is concave down.) **42**. a: $(0,\infty)$. b: none. c: $x = 0$. d: $(-\infty,-2)$ and $(2,\infty)$. e: $x = 2$ and $x = -2$. **43**. max = 4, min = -9/10. **45**. At the moment in question, balloon is rising .09 km/sec. **46**. a: At 4pm, the distance between the ships grows exactly 3.6 mi/hr. b: At 1pm, the distance between the ships grows exactly 2.5 mi/hr.

47. $\sqrt{6}$ (when you're at the point $(1,\sqrt{5})$ **48**. 32 in^2 **49**. $f(x) = 3 + 3x + \ln|x|$ **50**. $h(t) = -16t^2 - 4t + 170$ **51**. a: $10(6-2-1)$ (or 30). b: $5(6+3-2-3-1+1)$ (or 20). **52**. a: 0. b: 0. c: 1. d: -1. e: e^5. f: e.