

1 (8 pts). Sketch the graph of the function. Your graph doesn't need to be perfect, but it should clearly show the locations of any asymptotes and x -intercepts.

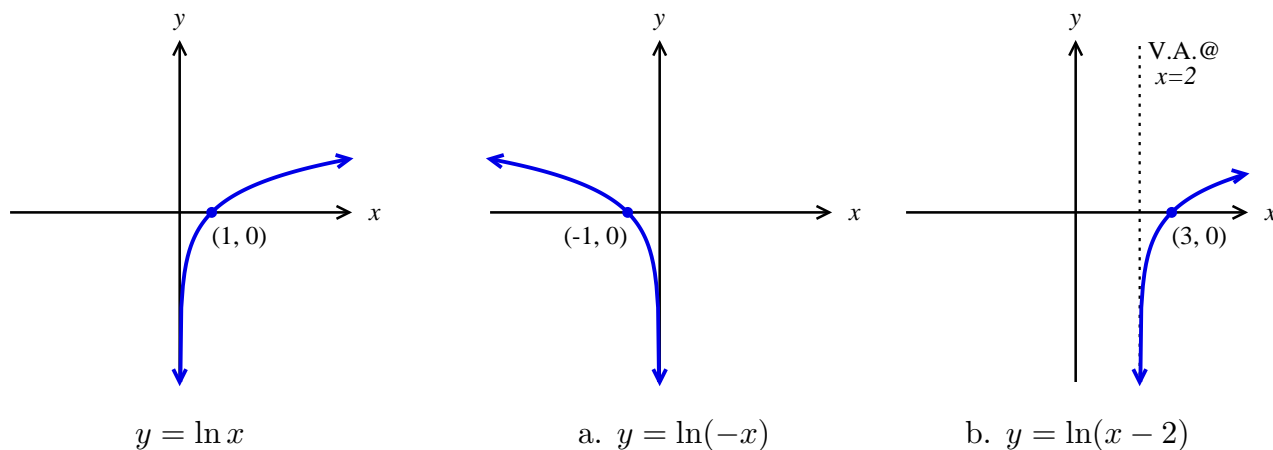
a. $y = \ln(-x)$

b. $y = \ln(x - 2)$

2 (2 pts). Evaluate the limit, if it exists: $\lim_{x \rightarrow 2^+} \ln(x - 2)$

Solution:

1.(Source: 1.5.47.48) Both of these graphs can be obtained from the graph of $y = \ln x$ (below left). In Part a, the graph of $y = \ln(-x)$ is obtained by reflecting the graph of $\ln x$ across the line $x = 0$. In Part b, the graph of $y = \ln(x - 2)$ is obtained by shifting the graph of $\ln x$ to the right 2 units.



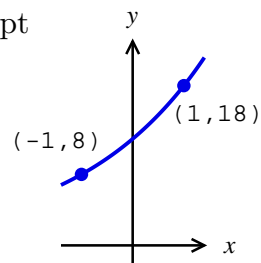
In both of these, it helps to think of where the functions are defined. Since $\ln x$ is defined only for $x > 0$, the function $\ln(-x)$ is defined only when $-x > 0$, or $x < 0$, and the function $\ln(x - 2)$ is defined only when $x - 2 > 0$, or $x > 2$.

2.(Source: 2.2.35) $(x - 2) \rightarrow 0^+$ as $x \rightarrow 2^+$, and so $\ln(x - 2) \rightarrow -\infty$. That is,

$$\lim_{x \rightarrow 2^+} \ln(x - 2) = -\infty.$$

This infinite limit is the reason for the vertical asymptote we saw in the graph of $\ln(x - 2)$ in Problem 1.

1 (10 pts). Find the constants C and $b > 0$ so that the exponential function $f(x) = Cb^x$ has the graph shown here. What is the y -intercept of this graph?



Solution:

1 (10 pts)(Source: 1.4.21-22) . From the given points obtain the equations

$$(-1, 8) \longrightarrow Cb^{-1} = 8$$

$$(1, 18) \longrightarrow Cb^1 = 18$$

Solve for C in the first to obtain $C = 8b$. Substitute this into the second and solve for b :

$$8b^2 = 18$$

$$b^2 = \frac{18}{8} = \frac{9}{4}$$

Since $b > 0$,

$$b = \sqrt{\frac{9}{4}} = \frac{3}{2}$$

and

$$C = 8b = 8 \cdot \frac{3}{2} = 12.$$

That is, $f(x) = 12 \left(\frac{3}{2}\right)^x$.

Since $f(0) = C = 12$, the y -intercept is the point $(0, 12)$.

(done)