MATH 120–03 (Kunkle), Quiz 6 | Name: ______________________
10 pts, 10 minutes | Nov 17, 2016

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1 (10 pts). A rectangular cardboard box with a square top and base will constructed using single thickness cardboard for the sides and double thickness cardboard for the base and top. If the box is to be made from 48 square inches of cardboard, what is the maximum possible volume of the box?

Solution:

1.(Source: 4.7.15-16) Let x be the sidelength of the base and y be the height of the box. Then the volume of the box is $V = x^2y$.

The total area of cardboard used in the box is the area of the 4 sides plus 2 times the areas of the base and of the top, and this must equal 48 in^2:

$$48 = 4x^2 + 4xy, \text{ or } 12 = x^2 + xy.$$

Solving,

$$y = \frac{12 - x^2}{x}$$

and so

$$V = x^2\left(\frac{12 - x^2}{x}\right) = x(12 - x^2) = 12x - x^3.$$

We want to maximize V, but over what interval? Obviously, $x > 0$ and $y > 0$. The largest x occurs when y is smallest, that is,

$$y = \frac{12 - x^2}{x} = 0$$

and this occurs when $x = \sqrt{12}$. Therefore, we to maximize $V(x)$ over the interval $0 < x \leq \sqrt{12}$. Volume is zero at both endpoints, so the max must occur at a critical point in the interior of the interval.

To find the critical point, set

$$0 = \frac{dV}{dx} = 12 - 3x^2 = 3(2 - x)(2 + x),$$

so $x = \pm 2$. Since x must be positive, 2 is the only relevant critical point, and the maximum volume is

$$V(2) = 12 \cdot 2 - 2^3 = 16.$$