1 (10 pts). Differentiate with respect to $x$:

a. $f(x) = (x^3 - 9x)e^x$

b. $g(x) = x^2 \cos x \sin x$

c. $h(x) = \tan(\sec^2 x)$

Solution:

1a. (Source: 3.2.3) $f$ is the product of two functions, so use the product rule:

$$f'(x) = [(x^3 - 9x)e^x]' = (x^3 - 9x)'e^x + (x^3 - 9x)(e^x)'$$

$$= (3x^2 - 9)e^x + (x^3 - 9x)e^x$$

$$= (x^3 + 3x^2 - 9x - 9)e^x$$

1b. (Source: 3.4.15) $g$ is the product of three functions.

$$g'(x) = (x^2)' \cos x \sin x + x^2(\cos x)' \sin x + x^2 \cos x(\sin x)'$$

$$= 2x \cos x \sin x + x^2(-\sin x) \sin x + x^2 \cos x(\cos x)$$

$$= 2x \cos x \sin x - x^2 \sin^2 x + x^2 \cos^2 x$$

1c. (Source: 3.4.37) $h$ is the composition of tan and $\sec^2$, not the product. Use the chain rule. Working from the outside in, first differentiate the tan function, then differentiate the squaring function, and last of all differentiate the sec function.

$$h'(x) = \tan'(\sec^2 x) \cdot 2(\sec x)^1(\sec' x)$$

$$= \sec^2(\sec^2 x)2 \sec x \sec x \tan x$$

$$= 2 \sec^2(\sec^2 x) \sec^2 x \tan x$$

Correction: The solutions I passed out in class were missing the 2 in the last line. Thanks to Rachel for pointing out my error.