

1 (10 pts). Differentiate with respect to x :

a. $f(x) = (x^3 - 9x)e^x$ b. $g(x) = x^2 \cos x \sin x$ c. $h(x) = \tan(\sec^2 x)$

Solution:

1a. (Source: 3.2.3) f is the product of two functions, so use the product rule:

$$\begin{aligned} f'(x) &= [(x^3 - 9x)e^x]' = (x^3 - 9x)'e^x + (x^3 - 9x)(e^x)' \\ &= (3x^2 - 9)e^x + (x^3 - 9x)e^x \\ &= (x^3 + 3x^2 - 9x - 9)e^x \end{aligned}$$

1b. (Source: 3.4.15) g is the product of three functions.

$$\begin{aligned} g'(x) &= (x^2)' \cos x \sin x + x^2(\cos x)' \sin x + x^2 \cos x(\sin x)' \\ &= 2x \cos x \sin x + x^2(-\sin x) \sin x + x^2 \cos x(\cos x) \\ &= 2x \cos x \sin x - x^2 \sin^2 x + x^2 \cos^2 x \end{aligned}$$

1c. (Source: 3.4.37) h is the composition of \tan and \sec^2 , not the product. Use the chain rule. Working from the outside in, first differentiate the \tan function, then differentiate the squaring function, and last of all differentiate the \sec function.

$$\begin{aligned} h'(x) &= \tan'(\sec^2 x) \cdot 2(\sec x)^1(\sec' x) \\ &= \sec^2(\sec^2 x) 2 \sec x \sec x \tan x \\ &= 2 \sec^2(\sec^2 x) \sec^2 x \tan x \end{aligned}$$

Correction: The solutions I passed out in class were missing the 2 in the last line. Thanks to Rachel for pointing out my error.