MATH 120–03 (Kunkle), Quiz 2 | Name: ____________________
10 pts, 10 minutes | Sept 8, 2016

1a (4 pts). State the precise, ε-δ definition of what it means for $\lim_{x\to a} f(x)$ to equal L.

1b (6 pts). Write an ε-δ proof of the fact that $\lim_{x\to 2}(3-5x) = -7$.

Solution:

(Source: 2.4.17)

1a. $\lim_{x\to a} f(x) = L$ means that for every positive number ε, there's a corresponding positive number δ with the property that $|f(x) - L| < \varepsilon$ whenever $0 < |x-a| < \delta$.

1b. To come up with proof, it helps to do some analysis first. Given $\varepsilon > 0$, we're looking for a number δ so that $0 < |x-2| < \delta$ will imply that $|(3-5x)-(-7)| < \varepsilon$. Start by simplifying:

$$|(3-5x)-(-7)| = |3-5x+7| = |10-5x| = |-5(x-2)| = |-5||x-2| = 5|x-2|,$$

so to make $5|x-2| < \varepsilon$, just make sure that $|x-2| < \frac{1}{5}\varepsilon$.

Now we're ready to write a proof.

Proof: Suppose that $\varepsilon > 0$. Choose $\delta = \frac{1}{5}\varepsilon$. Then, whenever

$$|x-2| < \delta = \frac{1}{5}\varepsilon,$$

$$|(3-5x)-(-7)| = |3-5x+7| = |10-5x| = |-5(x-2)| = |-5||x-2| = 5|x-2| < 5\cdot\frac{1}{5}\varepsilon = \varepsilon,$$

as desired. **(done)**