No notes, books, electronic devices, or outside materials of any kind.

Unless otherwise indicated, supporting work will be required on every problem worth more than 2 points.

Read each problem carefully and simplify your answers. You are expected to know the values of all trigonometric functions at multiples of $\pi/4$ and of $\pi/6$.

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1 (8 pts). Sketch the graph of a function $q(x)$ that satisfies all of the following:

- $q'(x) < 0$ for all $x \neq -1$.
- $x = -1$ is a vertical asymptote to the curve $y = q(x)$.
- $q''(x) > 0$ if $-1 < x < 2$.
- $q''(x) < 0$ if $x < -1$ or $x > 2$.

2 (10 pts). Find all critical points of $f(x) = x^{2/3}(x-2)$.

3 (14 pts). Let $g(x) = 4x + \dfrac{1}{x}$.

a. Find all x-values, if any, at which $g(x)$ is a local maximum.

b. Find all x-values, if any, at which $g(x)$ is a local minimum.

You are not required to tell me these maximum or minimum values of $g(x)$.

4a (15 pts). On what interval(s) is $h(x) = 4x^3 - 3x^2 - 6x$ decreasing? On what interval(s) is $y = h(x)$ concave down?

4b (1 pts). At which x-values, if any, does $h(x)$ have an inflection point?

4c (8 pts). Find the absolute maximum and absolute minimum values of $h(x)$ on the interval $[-2, 0]$.

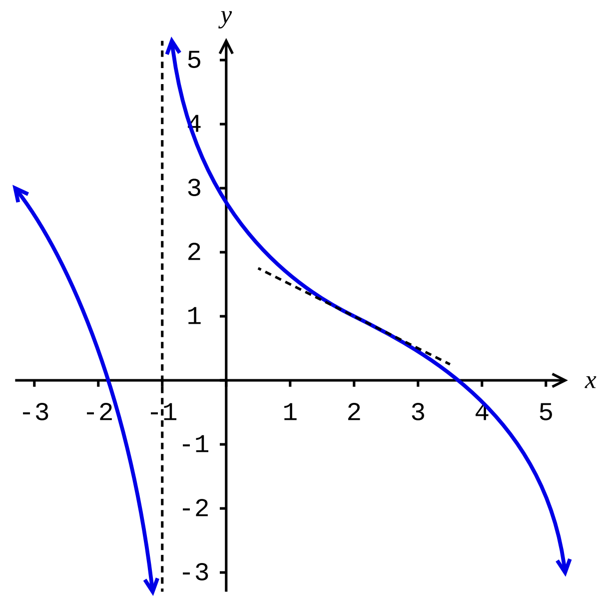
5 (19 pts). Evaluate the following limits.

a. $\displaystyle\lim_{x\to 0} \frac{\cos(3x) - \cos(5x)}{x^2}$ b. $\displaystyle\lim_{x\to\infty} x^{1/3}e^{-2x}$

6 (12 pts). Find the equations of all horizontal and vertical asymptotes of the curve.

a. $y = x^{1/3}e^{-2x}$ b. $y = \dfrac{2x^2+1}{x^2-16}$

7 (13 pts). Find all numbers c that satisfy the conclusion of the Mean Value Theorem as applied to the function $p(x) = x - 2\sin x$ on the interval $[0, \pi]$.



1.(Source: 4.3.28) The graph at the right in one correct solution, but there are others. For instance, $y = q(x)$ could have a horizontal asymptote as $x \to -\infty$.

$q(x)$ is always decreasing since $q'(x)$ is always negative (when it exists). The graph could have a horizontal asymptote to the left, but nothing in the problem says that it must. There's an inflection point at $x = 2$. Note that the tangent line at $x = 2$ (which you were not required to include in your graph) crosses the curve. This is because $q'(x)$ has a local max at $x = 2$.

The function $q(x) = \dfrac{1}{x+1} - e^{(x-4.6)}$ (approximately) has a graph as described in this problem, as does $\frac{1}{x+1} - e^{(x-4.6)} - ax$ for any positive number a.

2.(Source: 4.1.39) Critical points are those x-values at which $f(x)$ is defined but $f'(x)$ is either zero or undefined, so we calculate $f'(x) = \left(x^{2/3}(x-2)\right)' = \frac{2}{3}x^{-1/3}(x-2) + x^{2/3}$. The negative exponent means that f' does not exist at $x = 0$. To find the zeros of f', it helps to factor $f'(x)$:

$$x^{-1/3}(\tfrac{2}{3}(x-2)+x) = x^{-1/3}(\tfrac{5}{3}x-\tfrac{4}{3}) = 0 \Rightarrow x^{-1/3} = 0(\text{no solutions}) \text{ or } \tfrac{5}{3}x-\tfrac{4}{3} = 0 \Rightarrow x = \tfrac{4}{5}$$

So the critical points are $x = 0$ and $x = \frac{4}{5}$.

(You could solve the equation $f'(x) = 0$ without factoring. Below, I multiplied both sides of the equation by $3x^{1/3}$:

$$\tfrac{2}{3}x^{-1/3}(x-2) + x^{2/3} = 0 \;\Rightarrow\; 0 = 2(x-2) + 3x = 5x - 4 \;\Rightarrow\; x = \tfrac{4}{5} \text{ again.)}$$

3.(Source: 4.1.53, 4.3.12) Local extrema of $g(x) = 4x + \dfrac{1}{x} = 4x + x^{-1}$ can occur only at critical points, i.e., those points where $g'(x) = 4 - x^{-2} = 4 - \frac{1}{x^2}$ is undefined or zero. g' fails to exist only at $x = 0$, where g is also undefined, so local extrema can only occur where

$$4 - \frac{1}{x^2} = 0 \Rightarrow x^2 = \frac{1}{4} \Rightarrow x = \pm\frac{1}{2}.$$

Apply the second derivative test at these points: $g''(x) = 2x^{-3}$, so $g(1/2) = (1/2)^{-3} = (2)^3 = 8$ and $g(-1/2) = (-1/2)^{-3} = (-2)^3 = -8$. Because g is concave up at $x = 1/2$, $g(1/2)$ is a local minimum, and because g is concave down at $x = -1/2$, $g(-1/2)$ is a local maximum. (As explained in the statement of the Problem 3, you were not required to calculate these g-values.)

4a.(Source: 4.3.9) Make a sign chart for $h'(x) = 12x^2 - 6x - 6 = 6(2x+1)(x-1)$ and $h''(x) = 24x - 6 = 6(4x-1)$.

$6(4x-1)$:	$- - - - - -$	$-$	$- - - - - - - -$	0	$+ + + + + + +$	$+$	$+ + + + + +$
$6(2x+1)(x-1)$:	$+ + + + + +$	0	$- - - - - - - -$	$-$	$- - - - - - -$	0	$+ + + + + +$
x :		$-\frac{1}{2}$		$\frac{1}{4}$		1	

h decreases when $h'(x) < 0$, on $(-\frac{1}{2}, 1)$ and is concave down when $h''(x) < 0$, on $(-\infty, \frac{1}{4})$.

4b. $h(x)$ has an inflection point at $x = \frac{1}{4}$, where $h''(x)$ changes sign.

4c.(Source: 4.1.51) The absolute extrema can only occur at an endpoint or a critical point inside $(-2, 0)$, so just test $h(x)$ at $x = -2, -1/2$, and 0:

$$h(-2) = -32 \qquad h(-1/2) = 7/4 \qquad h(0) = 0$$

The absolute maximum is $7/4$ and the minimum is -32.

5a.(Source: 4.4.34) $\lim_{x\to 0} \frac{\cos(3x)-\cos(5x)}{x^2} =$ "$\frac{0}{0}$". l'Hôpitize: $\lim_{x\to 0} \frac{-3\sin(3x)+5\sin(5x)}{2x} =$ "$\frac{0}{0}$". l'Hôpitize again: $\lim_{x\to 0} \frac{-9\cos(3x)+25\cos(5x)}{2} = \frac{-9+25}{2} = 8$, so, by two applications of l'Hôpital's Rule, the original limit also equals 8.

5b.(Source: 4.4.44, 47) Write the product as a quotient: $\lim_{x\to\infty} x^{1/3}e^{-2x} = \lim_{x\to\infty} \frac{x^{1/3}}{e^{2x}} =$ "$\frac{\infty}{\infty}$", so we can try l'Hôpital's: $\lim_{x\to\infty} \frac{\frac{1}{3}x^{-2/3}}{2e^{2x}} = \lim_{x\to\infty} \frac{1}{6x^{2/3}e^{2x}} = \frac{1}{\infty} = 0$, so the original limit must also equal 0.

6a.(Source: 4.3.53, 4.4.44, 4.5.43) By Problem 5b, $y = 0$ is a horizontal asymptote of $y = x^{1/3}e^{-2x}$ as $x \to \infty$. (As $x \to -\infty$, $x^{1/3}e^{-2x} \to -\infty \cdot \infty = -\infty$, so the curve as no horizontal asymptote on the left. y is continuous on $(-\infty, \infty)$ and therefore has no vertical asymptotes.)

6b.(Source: 4.5.11) As $x \to \pm\infty$, $y = \frac{2x^2+1}{x^2-16}$ has the same limit as $y = \frac{2x^2}{x^2} = 2$, so $y = 2$ is a horizontal asymptote.

Vertical asymptotes occur at $x = 4$ and $x = -4$, since at these x-values, $\frac{2x^2+1}{x^2-16}$ is $\frac{\text{nonzero}}{0}$, indicating that $y \to \pm\infty$.

7.(Source: 4.2.7) Seek c in $(0, \pi)$ for which

$$p'(c) = 1 - 2\cos c = \frac{p(\pi) - p(0)}{\pi - 0} = \frac{(\pi - 2\sin\pi) - (0 - 2\sin 0)}{\pi} = \frac{(\pi - 0) - (0)}{\pi} = 1,$$

or $\cos c = 0$. The only solution on $(0, \pi)$ is $c = \pi/2$.