No notes, books, electronic devices, or outside materials of any kind.

Read each problem carefully and simplify your answers. Unless otherwise indicated, supporting work will be required on every problem worth more than 2 points.

You are expected to know the values of all trigonometric functions at multiples of $\pi/4$ and of $\pi/6$.

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1 (3 pts). State the derivative of each function.

a. $\sin x$	b. $\tan x$	c. $\sec x$
d. $\cos x$	e. $\cot x$	f. $\csc x$
g. $\sin^{-1} x$	h. $\tan^{-1} x$	i. $\sec^{-1} x$

2 (6 pts). Find $\frac{dy}{dx}$ along the curve $x^2 - 5xy + y^2 = x \ln y$.

3 (3 pts). Find the linearization $L(x)$ of the function $f(x) = x^{1/3}$ at $a = 27$.

4 (6 pts). The height above ground in meters of a projectile shot vertically upward from a point 3 m above ground level with an initial velocity of 4 m/sec is $h = -4t^2 + 4t + 3$ after t seconds.

a. Express the (vertical) velocity of the projectile as a function of t.

b. When does the projectile reach its maximum height?

c. When does the projectile hit the ground, and what is its velocity at that time? Label your answers with the correct units.

5 (6 pts). A dog and a horse start running from the same point. The dog runs north at 3 m/sec (meters per second) and the horse runs east at 4m/sec. At what rate is the distance between the two animals increasing 5 seconds later?

1 (1 pts). State the derivative of each function.

a. $(\sin x)' = \cos x$ b. $(\tan x)' = \sec^2 x$ c. $(\sec x)' = \sec x \tan x$

d. $(\cos x)' = -\sin x$ e. $(\cot x)' = -\csc^2 x$ f. $(\csc x)' = -\csc x \cot x$

g. $(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}$ h. $(\tan^{-1} x)' = \frac{1}{1+x^2}$ i. $(\sec^{-1} x)' = \frac{1}{|x|\sqrt{x^2-1}}$

(Also acceptable: $\frac{1}{x\sqrt{x^2-1}}$)

2. (Source: 3.5.5,11) Think of y as an unspecified function of x, differentiate both sides of $x^2 - 5xy + y^2 = x \ln y$ with respect to x, and solve for $\frac{dy}{dx}$ (here denoted y'):

$$2x - 5y - 5xy' + 2yy' = 1 \ln y + x\frac{1}{y}y'$$

$$2x - 5y - \ln y = xy^{-1}y' + 5xy' - 2yy' = (xy^{-1} + 5x - 2y)y'$$

$$\frac{2x - 5y - \ln y}{xy^{-1} + 5x - 2y} = y'$$

3. (Source: 3.10.3) $L(x) = f(a) + f'(a)(x - a)$. In this case, since $f(a) = 27^{1/3} = 3$, and $f'(a) = (1/3)27^{-2/3} = \frac{1}{3 \cdot 3^2} = 1/27$, so $L(x) = 3 + \frac{1}{27}(x - 27)$.

You were not required to rewrite this in $mx + b$ form, but if you did, then $L(x) = 2 + \frac{1}{27}x$.

4a. (Source: 3.7.7) Height $h = -4t^2 + 4t + 3$ m, so velocity $v = h' = -8t + 4$ m/sec.

4b. The short answer: when $v = -8t + 4 = 0$, or $t = 1/2$ sec. Here's some supporting work, not all of which is necessary for full credit.

Here's a sign chart for v:

$-8t + 4:$	$+ + + + + + 0 - - - - - - - -$	
$t:$	0	1/2

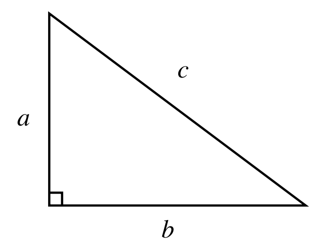
The projectile is rising for $0 \le t \le 1/2$ and falling afterwards, so its maximum height is at $t = 1/2$ sec.

4c. The projectile hits the ground when $h = 0$. Solving,

$$0 = -4t^2 + 4t + 3 = (-2t + 3)(2t + 1) \Rightarrow t = 3/2 \text{ or } -1/2.$$

Since our formula for h is valid only for $t \ge 0$, the projectile hits the ground at $t = 3/2$ sec. At that time, its velocity is $v = -8(3/2) + 4 = -8$ m/sec.

5 (1 pts). (Source: 3.9.17) See figure. Let a and b stand for the distance traveled by the dog and horse, respectively, and c the distance between the two animals. The problem gives $\frac{da}{dt}$ and $\frac{db}{dt}$ and asks for $\frac{dc}{dt}$ at one particular moment.



To find a relation between these three derivatives, start with the Pythagorean relation between a, b, and c:

$$a^2 + b^2 = c^2$$

This equation is true for all time and not just at one particular moment. This allows us to differentiate both sides with respect to time t:

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

Divide both sides by 2 and substitute $\frac{da}{dt} = 3$ and $\frac{db}{dt} = 4$:

$$3a + 4b = c\frac{dc}{dt}$$

After five seconds, $a = 15$ and $b = 20$, and, by Pythagorus, $c = 25$:

$$3 \cdot 15 + 4 \cdot 20 = 25\frac{dc}{dt}$$

Solving, we find $\frac{dc}{dt} = (45 + 80)/25 = 125/25 = 5$ m/sec.

5. **Alternate solution.** Since the dog and horse move at constant speeds,

$$a = 3t \text{ and } b = 4t,$$

where t is the number of seconds after they both start. Therefore

$$c = \sqrt{a^2 + b^2} = \sqrt{(3t)^2 + (4t)^2} = \sqrt{9t^2 + 16t^2} = \sqrt{25t^2} = 5t.$$

(We can take the positive square root in the last step above because c and t are both positive in this problem.) Now we can calculate $\frac{dc}{dt} = 5$ directly. In fact, $\frac{dc}{dt}$ is 5 m/sec for all time, not just at $t = 5$ sec.