Find the absolute maximum and absolute minimum of the function \( f(x) = \frac{x}{x^2 + 4} \) on the interval \([0, 4]\).

**Solution.** The max and min must occur at either the endpoints or critical points in \((0, 4)\).

Search for critical points:

\[
f'(x) = \frac{1(x^2 + 4) - x \cdot 2x}{(x^2 + 4)^2} = \frac{x^2 + 4 - 2x^2}{(x^2 + 4)^2} = \frac{4 - x^2}{(x^2 + 4)^2}.
\]

\( f'(x) \) exists for all real \( x \), since \( x^2 + 4 \) has no real zeros. Set \( f'(x) = 0 \) and solve for \( x \):

\[
0 = \frac{4 - x^2}{(x^2 + 4)^2} \Rightarrow 0 = 4 - x^2 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2.
\]

Of these, only \( x = 2 \) is in \((0, 4)\).

Evaluate and compare \( f \) at \( x = 0, 2, \sqrt{4} \).

\[
f(0) = \frac{0}{0^2 + 4} = 0
\]

\[
f(2) = \frac{2}{4 + 4} = \frac{1}{4}
\]

\[
f(4) = \frac{4}{16 + 4} = \frac{4}{20} = \frac{1}{5}
\]

The largest of this, \( \frac{1}{5} \), is the absolute max of \( f \). The smallest, 0, is the absolute min.