MATH 120–04 (Kunkle), Exam 4 Name: ______________________

100 pts, 75 minutes Nov. 18, 2014 Page 1 of 1

No notes, books, electronic devices, or outside materials of any kind.

Read each problem carefully and simplify your answers.

Source Supporting work will be required on every problem worth more than 2 points.

4.4 1 (17 pts). Evaluate the limits.

a. $\lim\limits_{x\to 0} \dfrac{e^{3x}-1}{x}$ b. $\lim\limits_{t\to 0} \dfrac{\cos t}{1+\sin t}$ c. $\lim\limits_{x\to\infty} \left(1-\dfrac{5}{x}\right)^{x}$

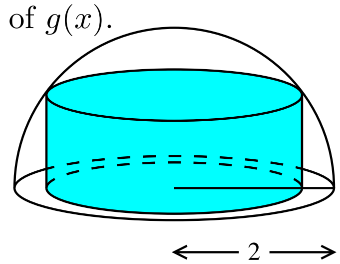
4.5 2 (10 pts). This problem is about the function $g(x) = \dfrac{6}{1-e^{-x}}$ and its graph.
(Some supporting work is required for each part.)

a. What is the domain of $g(x)$?

b. Find the equations of all vertical asymptotes of the graph of $g(x)$.

c. Find the equations of all horizontal asymptotes of the graph of $g(x)$.

4.7 3 (16 pts). A cylinder is inscribed in a hemisphere of radius 2. Find the largest possible volume of the cylinder. (Don't worry about simplifying your answer to this problem.)



5.1, 5.2 4 (6 pts). The velocity of an object moving along a coordinate axes is $v(t) = \sin(t^2)$ m/sec at time t sec. Approximate the displacement of the object between times $t = 2$ and $t = 10$ with a Riemann sum using $n = 4$ subintervals and their midpoints.
Your answer should contain only numbers and arithmetic operations. No "..." and no "$f(x)$" allowed. You can leave unfinished arithmetic in your answer.

5.3 5 (10 pts). Find the derivative: $\dfrac{d}{dx}\displaystyle\int_{x^2}^{1} e^{(t^4)}\,dt$

5.2 6 (9 pts). Find the following, if $\int_{-2}^{5} h(x)\,dx = 8$ and $\int_{1}^{-2} h(x)\,dx = 5$.

a. $\int_{1}^{5} h(x)\,dx =$ b. $\int_{-2}^{1} 4h(x)\,dx =$ c. $\int_{-2}^{5} \big(2+h(x)\big)\,dx =$

5.2, 5.3 7 (16 pts). Evaluate the definite integral.

a. $\displaystyle\int_{-5}^{-2} \frac{1}{t}\,dt$ b. $\displaystyle\int_{0}^{1} \frac{5}{1+x^2}\,dx.$ c. $\displaystyle\int_{-2}^{4} |x|\,dx$

4.9, 5.4 8 (16 pts). Find the general antiderivative of the given function.

a. $f(x) = (2+x)(5-x^2)$ b. $f(x) = 4\sec x\tan x - 2\sec^2 x + e^x$

c. $f(x) = 3\sin x + 5\cos x + \dfrac{1}{\sqrt{1-x^2}}$

4 PTS 1a. $\lim_{x\to 0}\frac{e^{3x}-1}{x} = "\frac{0}{0}"$. l'Hospitalize: $\lim_{x\to 0}\frac{3e^{3x}}{1} = 3$, so $\lim_{x\to 0}\frac{e^{3x}-1}{x}$ also $= 3$.

Alternate form of this question: replace 3 by 2 throughout.

3 PTS 1b. $\lim_{t\to 0}\frac{\cos t}{1+\sin t} = \frac{1}{1+0} = 1$. (l'Hospital's Rule does not apply.)

10 PTS 1c. 1^{∞}. Let $y = (1-\frac{5}{x})^x$. Then $\ln y = \ln(1-\frac{5}{x})^x = x\ln(1-5/x)$

$= \frac{\ln(1-5x^{-1})}{x^{-1}}$ "$\frac{0}{0}$". l'Hospitalize: $\frac{\frac{1}{1-5x^{-1}}\cdot(-5)(-x^{-2})}{(-x^{-2})} \to \frac{-5}{1-0} = -5$.

By l'Hospital's Rule, $\ln y$ also $\to -5$.

Therefore, $y = e^{\ln y} \to e^{-5}$. Alt. form: Replace 5 by 3 throughout.

2 PTS 2a. For $g(x)$ to exist, need denominator $\neq 0$. $1-e^{-x}\neq 0$. $e^{-x}\neq 1$. $x\neq 0$.

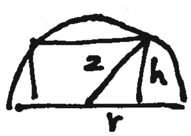
Domain = all real numbers except 0; (or $(-\infty,0)\cup(0,\infty)$, not required)

2 PTS 2b. $g(x)$ blows up @ $x=0$, because $g(0) = "\frac{0}{0}"$. VA is $\underline{x=0}$.

6 PTS 2c. $\lim_{x\to\infty} g(x) = \lim_{x\to\infty}\frac{6}{1-e^{-x}} = \frac{6}{1-0} = 6$. $y=6$ is HA. (alt: $y=4$)

$\lim_{x\to-\infty} g(x) = \lim_{x\to-\infty}\frac{6}{1-e^{-x}} = "\frac{6}{1-\infty}" = 0$. $y=0$ is another HA.

3.



Maximize Vol $= \pi r^2 h$ subject to constraint $r^2+h^2=2^2$.

$r^2 = 4-h^2$, $V = \pi(4-h^2)h = \pi(4h-h^3)$. $V' = \pi(4-3h^2)$.

$V'=0$ @ $h = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}$. Limits on h: $0\le h\le 2$.

$V(0)=0$, $V(2)=0$, $V(2/\sqrt{3}) = \pi(4-\frac{4}{3})\frac{2}{\sqrt{3}} = \frac{16\pi}{3\sqrt{3}}$ = maximum volume.

4. t-interval: 2 4 6 8 10. midpoints = 3, 5, 7, 9.

RS $= \Delta t(f(3)+f(5)+f(7)+f(9)) = 2(\sin 9 + \sin 25 + \sin 49 + \sin 81)$.

5. Let $I = \int_{x^2}^{1} e^{t^4}dt = -\int_1^{x^2} e^{t^4}dt$, and let $u = x^2$. By FTC1,

$\frac{dI}{du} = -e^{(x^2)^4} = -e^{x^8}$. By Chain Rule, $\frac{dI}{dx} = \frac{dI}{du}\cdot\frac{du}{dx} = -2x\,e^{(x^8)}$.

Alt: t^3 replaces t^4. $\frac{dI}{dx} = -2x e^{(x^2)^3} = -2x e^{(x^6)}$.

2PTS 6a. $\int_1^5 h(x)\,dx = \int_1^{-2} h(x)\,dx + \int_{-2}^5 h(x)\,dx = 5+8 = 13$ (alt $= 7+4 = 11$).

3PTS 6b. $\int_{-2}^1 4h(x)\,dx = 4\int_{-2}^1 h(x)\,dx = -4\int_1^{-2} h(x)\,dx = -4\cdot 5 = -20$ (alt $= -4\cdot 7 = -28$).

4PTS 6c. $\int_{-2}^5 (2+h(x))\,dx = \int_{-2}^5 2\,dx + \int_{-2}^5 h(x)\,dx = 2(5-(-2)) + 8$ (Prop. 1, p. 373)

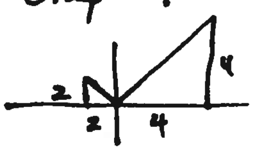
$= 2\cdot 7 + 8 = 22$ (alt $= 2\cdot 7 + 4 = 18$).

5PTS 7a. $\int_{-5}^{-2} \frac{1}{t}\,dt = \ln|t|\Big|_{-5}^{-2} = \ln|-2| - \ln|-5| = \ln 2 - \ln 5$ $(= \ln 2/5)$

Alt form: replace 2 by 3 throughout.

6PTS 7b. $\int_0^1 \frac{5}{1+x^2}\,dx = 5\tan^{-1}x\Big|_0^1 = 5(\tan^{-1}1 - \tan^{-1}0) = 5\left(\frac{\pi}{4} - 0\right) = \frac{5\pi}{4}$ (alt $\frac{3\pi}{4}$).

5PTS 7c. Graph $y = |x|$:



$\int_{-2}^4 |x|\,dx$ = total area of two triangles $= \frac12\cdot 2\cdot 2 + \frac12\cdot 4\cdot 4 = 10$

(alt $\frac12\cdot 2\cdot 2 + \frac12\cdot 6\cdot 6 = 20$)

8a. $(2+x)(5-x^2) = 10 + 5x - 2x^2 - x^3$. $F(x) = 10x + \frac52 x - \frac23 x^3 - \frac14 x^4 + C$.

8b. $f(x) = 4\sec x\tan x - 2\sec^2 x + e^x$. $F(x) = 4\sec x - 2\tan x + e^x + C$.

8c. $f(x) = 3\sin x + 5\cos x + \frac{1}{\sqrt{1-x^2}}$. $F(x) = -3\cos x + 5\sin x + \sin^{-1}x + C$

(or $-3\cos x + 5\sin x - \cos^{-1}x + C$)

Alt:

$f(x) = 3\cos x + 5\sin x + \frac{1}{\sqrt{1-x^2}}$. $F(x) = 3\sin x - 5\cos x + \sin^{-1}x + C$

Note. To check your answers in problem 8, differentiate F. Does $F'(x) = f(x)$?

F(x) in 8a, 8b, 8c, excluding "+C", worth 8 PTS, 3 PTS, 3 PTS.

+C on 8a, 8b, 8c worth 2 PTS altogether