1 (17 pts). Evaluate the limits.

a. \[ \lim_{x \to 0} \frac{e^{3x} - 1}{x} \]

b. \[ \lim_{t \to 0} \frac{\cos t}{1 + \sin t} \]

c. \[ \lim_{x \to \infty} \left(1 - \frac{5}{x}\right)^x \]

2 (10 pts). This problem is about the function \( g(x) = \frac{6}{1 - e^{-x}} \) and its graph.

(Some supporting work is required for each part.)

a. What is the domain of \( g(x) \)?

b. Find the equations of all vertical asymptotes of the graph of \( g(x) \).

c. Find the equations of all horizontal asymptotes of the graph of \( g(x) \).

3 (16 pts). A cylinder is inscribed in a hemisphere of radius 2.

Find the largest possible volume of the cylinder. (Don’t worry about simplifying your answer to this problem.)

4 (6 pts). The velocity of an object moving along a coordinate axes is \( v(t) = \sin(t^2) \) m/sec at time \( t \) sec. Approximate the displacement of the object between times \( t = 2 \) and \( t = 10 \) with a Riemann sum using \( n = 4 \) subintervals and their midpoints.

Your answer should contain only numbers and arithmetic operations. No “…” and no “\( f(x) \)” allowed. You can leave unfinished arithmetic in your answer.

5 (10 pts). Find the derivative: \[ \frac{d}{dx} \int_1^x e^{(t^4)} \, dt \]

6 (9 pts). Find the following, if \( \int_{-2}^5 h(x) \, dx = 8 \) and \( \int_{-2}^1 h(x) \, dx = 5 \).

a. \( \int_1^5 h(x) \, dx = \)

b. \( \int_{-2}^1 4h(x) \, dx = \)

c. \( \int_{-2}^5 (2 + h(x)) \, dx = \)

7 (16 pts). Evaluate the definite integral.

a. \( \int_{-5}^{-2} \frac{1}{t} \, dt \)

b. \( \int_0^1 \frac{5}{1 + x^2} \, dx. \)

c. \( \int_{-2}^4 |x| \, dx \)

8 (16 pts). Find the general antiderivative of the given function.

a. \( f(x) = (2 + x)(5 - x^2) \)

b. \( f(x) = 4 \sec x \tan x - 2 \sec^2 x + e^x \)

c. \( f(x) = 3 \sin x + 5 \cos x + \frac{1}{\sqrt{1 - x^2}} \)
4 pts 1a. \( \lim_{x \to 0} \frac{e^{3x} - 1}{x} = 0 \). L'Hopital's Rule: \( \lim_{x \to 0} \frac{3e^{3x}}{1} = 3 \), so \( \lim_{x \to 0} \frac{e^{3x} - 1}{x} = 3 \).

Alternate form of this question: replace 3 by 2 throughout.

3 pts 1b. \( \lim_{t \to 0} \frac{\cos t}{1 + 3t} = 1 \). (L'Hopital's Rule does not apply.)

10 pts 1c. \( x^2 \Rightarrow -x^2 \). Then \( \ln y = \ln \left(1 - \frac{x^2}{x^2}\right) = \ln \left(1 - \frac{1}{x^2}\right) \Rightarrow \frac{1}{x^2} \left(1 - \frac{1}{x^2}\right) \rightarrow \frac{-1}{0} = -\infty \).

By L'Hopital's Rule, \( \ln y \to -\infty \).

Therefore, \( y = e^{\ln y} \rightarrow e^{-\infty} = 0 \). Alt. form: Replace 5 by 3 throughout.

2 pts 2a. For \( g(x) \) to exist, need denominator \( \neq 0 \), \( 1 - e^x \neq 0 \), \( e^x \neq 1 \). \( x \neq 0 \).

Domain = all real numbers except 0; (or \( (-\infty, 0) \cup (0, \infty) \), not required)

2 pts 2b. \( g(x) \) blows up at \( x = 0 \), because \( g(0) = \frac{0}{0} \). VA is \( x = 0 \).

6 pts 2c. \( \lim_{x \to 0} g(x) = \lim_{x \to 0} \frac{6}{1 - e^x} = \frac{6}{1 - 0} = 6 \). y = 6 is HA. (alt: y = 4)

\( \lim_{x \to 0} g(x) = \lim_{x \to 0} \frac{6}{1 - e^x} = \frac{6}{1 - 0} = 6 \). y = 0 is another HA.

3. Maximize \( V = \pi r^2 h \) subject to constraint \( r^2 + h^2 = 2r \).

\( V = \pi (4 - h^2) h = \pi (4h - h^3) \), \( V' = \pi (4 - 3h^2) \).

\( V = 0 \) @ \( h = \sqrt{\frac{4}{3}} \). Limits: \( 0 \leq h \leq 2 \).

\( V(0) = 0 \), \( V(2) = 0 \), \( V'(\sqrt{\frac{4}{3}}) = \pi \left(4 - \frac{4}{3}\right) \frac{2}{\sqrt{3}} = \frac{16\pi}{9\sqrt{3}} = \text{maximum volume} \).

4. t-interval: \( [0, 0.5, 1] \). max points = 3, 5, 7, 9.

\( R_1 = \int (f(3) + f(5) + f(7) + f(9)) = 2(3m + 5m + 7m + 9m) \).

5. Let \( I = \int_{-1}^{1} e^{x^4} \, dx \) and let \( u = x^2 \). By FTC1,

\( \frac{du}{dx} = -e^{x^4} \). By Chain Rule, \( \frac{dI}{dx} = \frac{dI}{du} \frac{du}{dx} = -2x e^{x^4} \).

Alt: \( t^3 \) replaces \( t^4 \). \( \frac{dI}{dx} = -2xe^{x^3} = -2xe^{x^6} \).
2Prs 6a. \( \int_{-2}^{2} h(x) \, dx = \int_{-2}^{2} h(x) \, dx + \int_{-2}^{2} h(x) \, dx = 5 + 8 = 13 \) (alt: \( 7 + 4 = 11 \)).

3Prs 6b. \( \int_{-2}^{2} 4h(x) \, dx = 4 \int_{-2}^{2} h(x) \, dx = -4 \int_{-2}^{2} h(x) \, dx = -4 \cdot 5 = -20 \) (alt: \( -4 \cdot 7 = -28 \)).

4Prs 6c. \( \int_{-2}^{2} \left( 2 + h(x) \right) \, dx = \int_{-2}^{2} 2 \, dx + \int_{-2}^{2} h(x) \, dx = 2(5 - (-2)) + 8 \) (Prop. 1, p. 873)
\( = 2 \cdot 7 + 8 = 22 \) (alt: \( 2 \cdot 7 - 4 = 10 \)).

5Prs 7a. \( \int_{-5}^{5} \frac{1}{x} \, dx = \ln |x| \bigg|_{-5}^{5} = \ln 5 - \ln 5 = 0 \) (alt: \( 0 \)).

Att from 2 by 3 throughout.

6Prs 7b. \( \int_{0}^{1} \frac{x}{1 + x^2} \, dx = \frac{x}{2} \int_{0}^{1} \frac{1}{2} (1 - x^2) \, dx = 5 \left( \frac{1}{2} - 0 \right) = \frac{5}{4} \) (alt: \( \frac{5}{4} \)).

5Prs 7c. Graph \( y = 1/x \):

\[ \int_{2}^{4} 1/x \, dx = \text{total area of two triangles } = \frac{1}{2} \cdot 2 \cdot 2 + \frac{1}{2} \cdot 4 \cdot 4 = 10 \] (alt: \( \frac{1}{2} \cdot 2 \cdot 2 + \frac{1}{2} \cdot 4 \cdot 4 = 10 \)).

8a. \( (2 + x)(5 - x^2) = 10 + 5x - 2x^2 - x^3 \). \( F(x) = 10x + \frac{5}{2} x - \frac{2}{3} x^3 - \frac{1}{4} x^4 + C \).

8b. \( f(x) = 4 \sec x \tan x - 2 \sec^2 x + e^x \). \( F(x) = 4 \sec x - 2 \tan x + e^x + C \).

8c. \( f(x) = 3 \sin x + 5 \cos x \). \( F(x) = -3 \cos x + 5 \sin x + C \)

Att:
\( f(x) = 3 \cos x + 5 \sin x + \frac{1}{\sqrt{1 - x^2}} \). \( F(x) = 3 \sin x - 5 \cos x + \sin x + C \)

Note. To check your answers in problems 8, differentiate \( F \). Does \( F'(x) = f(x) \)?
\( F(x) \) in 8a, 8b, 8c, excluding \( +C \), worth 8 pts. 3 pts, 3 pts.
\( +C \) on 8a, 8b, 8c worth 2 pts altogether.