MATH 120–04 (Kunkle), Exam 3 Name: ______________________

100 pts, 75 minutes Oct. 21, 2014 Page 1 of 1

No notes, books, electronic devices, or outside materials of any kind.

Read each problem carefully and simplify your answers.

Source Supporting work will be required on every problem worth more than 2 points.

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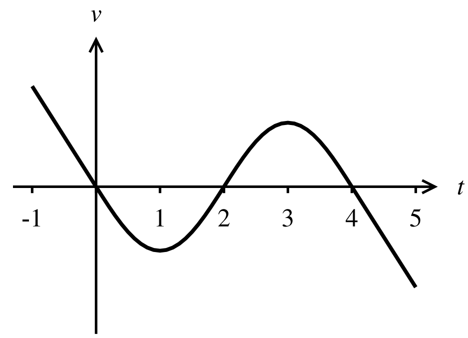
1 (9 pts). The accompanying graph is of the velocity $v(t)$ of a particle moving along an axis from time $t = -1$ to $t = 5$.



3.7 a. On what interval(s) of time is the particle moving in the positive direction?

4.1 b. At what time(s) does the particle's position attain a local minimum?

3.7 c. On what interval(s) of time is velocity decreasing?

3.7 d. On what interval(s) of time is the particle slowing down?

3.9 2 (15 pts). A rocket flies vertically away from its launchpad as an observer standing 1 km from the launchpad records the rocket's flight. How fast is the angle of elevation from the observer to the rocket increasing when the rocket is 2 km above the launchpad and traveling 3 km/sec? (Don't forget the units in your answer.)

4.1 3 (6 pts). On the axes provided, sketch the graph of a function $p(x)$ that's continuous on the open interval (1,5), has its absolute max at $x = 2$, a local min at $x = 4$, but no absolute min. Your function should not be defined at any x's outside of (1,5).

4.3 4 (20 pts). Let $q(x) = x + \frac{1}{x-2}$.

a. What is the domain of $q(x)$? State your answer in interval form.

b. On what interval(s) is $q(x)$ increasing?

c. On what interval(s) is $q(x)$ concave down?

3.10 5a (6 pts). Find dy if $y = e^{-\sin x}$.

3.10 5b (8 pts). Find the linearization of $f(x) = e^{-\sin x}$ at $a = 0$.

4.2 6 (12 pts). Find the number c guaranteed by the Mean Value Theorem when applied to the function e^{-x} on the interval $[-1, 1]$.

4.3 7 (9 pts). Sketch the graph of a function $h(x)$ satisfying all of the following conditions.

- $h(0) = 1$
- $h'(x) < 0$ if $x < 0$ or $0 < x < 3$
- $h''(x) < 0$ if $0 < x < 2$
- $h'(0) = h'(3) = 0$
- $h'(x) > 0$ if $x > 3$
- $h''(x) > 0$ if $x < 0$ or $x > 2$

4.1 8 (15 pts). Find the absolute maximum and absolute minimum of $f(x) = \frac{1}{2}x^4 - 3x^2$ on the interval $[0, 2]$.

1a. When $v > 0$, on $[-1,0) \cup (2,4)$. (Alt: when $v < 0$, on $(0,2) \cup (4,5]$.)

b. Sign chart for

v: + + 0 − − − − 0 + + + 0 − − −

t: −1 0 1 2 3 4 5

1^{st} der. Test $\Rightarrow$ local min @ $t = 2$

(alt: local max @ $t = 0$ and $t = 4$.)

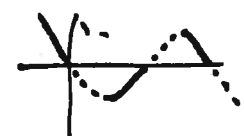
c. v decreasing on $[-1, 1]$ and $[3, 5]$. (alt: v inc. on $[1, 3]$.)

d. Either sketch speed $= |v(t)|$:

slowing down = speed decreasing : $[-1,0]$, $[1,2]$, $[3,4]$.

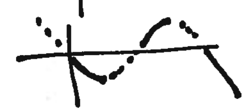
(speed up on $[0,1]$, $[2,3]$, $[4,5]$)

or slow down = velocity getting closer to 0 :



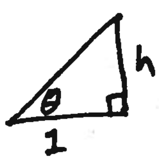
(same answers)

speed up = velocity getting farther from 0 :



(same answers)

2.

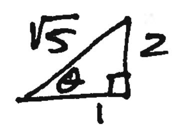


Given $\frac{dh}{dt} = 3$ (alt. 4). Want $\frac{d\theta}{dt}$ when $h = 2$.

Need equation relating h to θ: $\tan\theta = \frac{h}{1} = h$.

Differentiate w.r.t. t: $\sec^2\theta \frac{d\theta}{dt} = \frac{dh}{dt}$

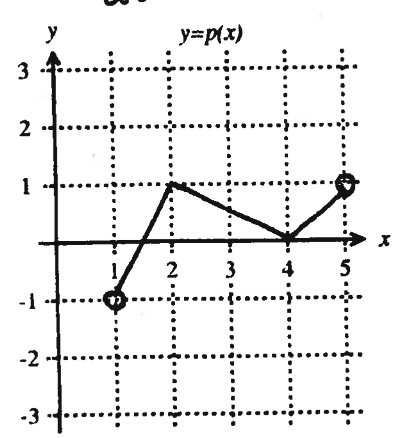
when $h = 2$, find hypotenuse by Pythagorus:



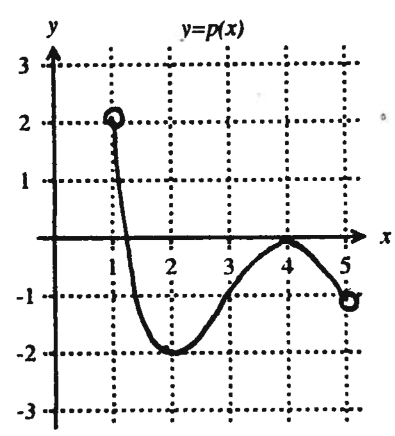
$\sec\theta = \frac{1}{\cos\theta} = \sqrt{5}$. Plug in & solve for $\frac{d\theta}{dt}$:

$5 \frac{d\theta}{dt} = 3 \Rightarrow \frac{d\theta}{dt} = \frac{3}{5}$ (alt $\frac{4}{5}$) radians per second.

3.



3. alt



4. a. domain q = all real numbers except 2: $(-\infty,2)\cup(2,\infty)$

(alt $(-\infty,3)\cup(3,\infty)$)

b. $q'(x) = 1 + \left(\frac{1}{x-2}\right)' = 1 + \left((x-2)^{-1}\right)' = 1-(x-2)^{-2}\cdot 1 = 1-\frac{1}{(x-2)^2}$

To make a sign chart for q', best to factor it; first subtract.

$q' = \frac{(x-2)^2-1}{(x-2)^2} = \frac{(x-2-1)(x-2+1)}{(x-2)^2} = \frac{(x-3)(x-1)}{(x-2)^2}$ sign change @ $x=1,3$, not @ $x=2$.

q': $+$ 0 $-$ DNE $-$ 0 $+$ (at $x = 1$, 2, 3)

q inc when $q'>0$, on $(-\infty,1)$ and $(3,\infty)$.

(alt: $+$ $-$ $-$ $+$ (at $x = 2$, 3, 4). $(-\infty,2)$ and $(4,\infty)$.)

c. $q' = 1-(x-2)^{-2}$, so $q'' = +2(x-2)^{-3} = \frac{2}{(x-2)^3}$.

q'': $-$ $-$ DNE $+$ $+$ (at $x = 2$). q conc. down when $q''<0$, on $(-\infty,2)$.

(alt: $(-\infty,3)$.)

5a. $y = e^{-\sin x} \Rightarrow dy = \frac{dy}{dx}dx = (e^{-\sin x})(-\sin x)'dx = -\cos x\, e^{-\sin x}dx$.

5b. At $a=0$, $y = e^{-\sin 0} = e^0 = 1$. $\frac{dy}{dx} = -\cos 0\, e^{-\sin 0} = -1$.

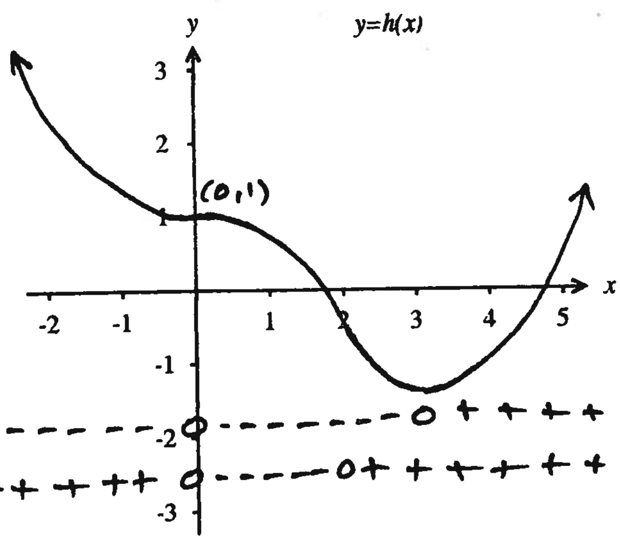
$L(x) = f(a) + f'(a)(x-a) = 1-1(x-0) = 1-x$.

6. (MVT says that if f is continuous on $[a,b]$ and differentiable on (a,b), then $f'(c) = (f(b)-f(a))/(b-a)$ for some c in (a,b).)

$f(x) = e^{-x}$. $f'(x) = -e^{-x}$. $-e^{-c} = \frac{e^{-1}-e^{-(-1)}}{1-(-1)} = \frac{e^{-1}-e}{2} \Rightarrow$

$e^{-c} = \frac{e-e^{-1}}{2} \Rightarrow -c = \ln\left(\frac{e-e^{-1}}{2}\right) \Rightarrow c = -\ln\left(\frac{e-e^{-1}}{2}\right)$.

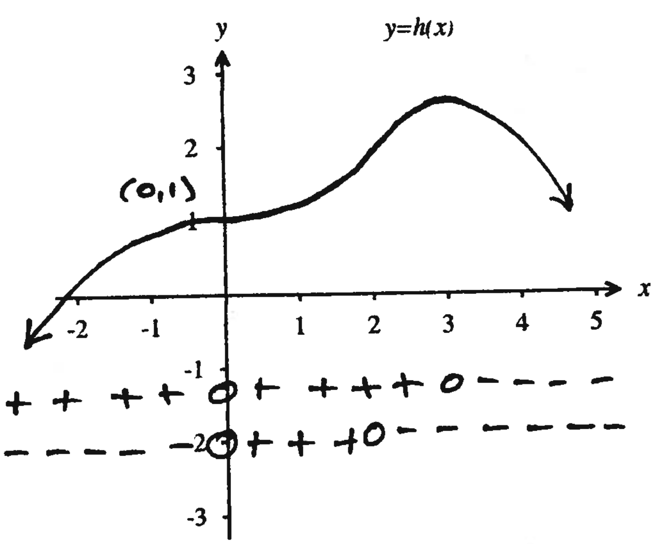
7.



h' $-$ $-$ $-$ $-$ $-$ $-$ 0 $-$ $-$ $-$ $-$ $-$ $-$ $-$ 0 $+$ $+$ $+$ $+$

h'' $+$ $+$ $+$ $+$ $+$ 0 $-$ $-$ $-$ $-$ 0 $+$ $+$ $+$ $+$ $+$ $+$

7. alt.



h' $+$ $+$ $+$ $+$ 0 $+$ $+$ $+$ $+$ 0 $-$ $-$ $-$ $-$

h'' $-$ $-$ $-$ $-$ $-$ 0 $+$ $+$ $+$ 0 $-$ $-$ $-$ $-$ $-$ $-$

8. Find critical points. $f' = 4\cdot\frac{1}{2}x^3 - 3\cdot 2x = 2x^3 - 6x = 2x(x^2-3)$

f' is always defined, so the only critical points are where $f'=0$.

$2x(x^2-3)=0$, $2x(x-\sqrt{3})(x+\sqrt{3})=0$ $x=0, \pm\sqrt{3}$.

Of these, only $\sqrt{3}$ is in the interval $(0,2)$.

Compute and compare:

$f(0) = 0$. $f(2) = \frac{1}{2}\cdot 2^4 - 3\cdot 2^2 = 8-12 = -4$.

$f(\sqrt{3}) = \frac{1}{2}(\sqrt{3})^4 - 3(\sqrt{3})^2 = \frac{1}{2}\cdot 3^2 - 3^2 = -\frac{9}{2} = -4.5$.

Absolute max $= 0$, and absolute min $= -4.5$.