MATH 120–04 (Kunkle), Exam 2 Name: ______________________

100 pts, 75 minutes Sept. 30, 2014 Page 1 of 1

No notes, books, electronic devices, or outside materials of any kind.

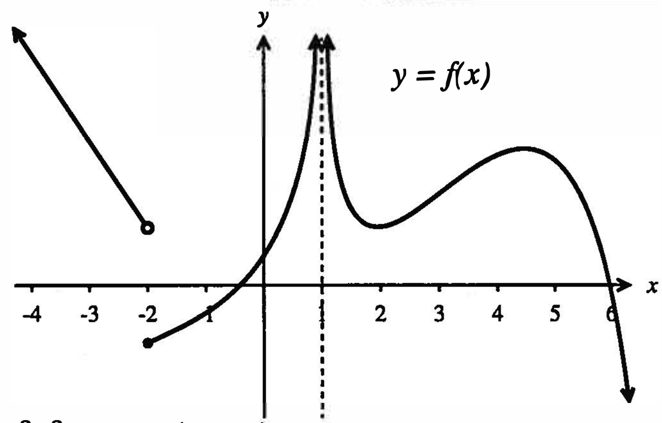
Read each problem carefully and simplify your answers. (Generally, this means to combine like terms, simplify exponents, add fractions that already have the same denominator, cancel common factors in a fraction, etc. Raise your hand if you're not sure whether to perform some simplification.)

Source Supporting work will be required on every problem worth more than 2 points.

2.8 1 (12 pts). The graph of the function $f(x)$ appears in the figure at right.

a. At which x-values does $f'(x)$ fail to exist?

b. Sketch the graph of $f'(x)$ on the axes provided.



3.1 2 (8 pts). Find $g'(x)$ and $g''(x)$ if $g(x) = xe^x$. Factor your answers.

3.3, 3.5 3a (14 pts). Find $\frac{dy}{dx}$ along the curve $x^2 + x^2y^2 = 2\cos(x+y)$.

3.5 3b (5 pts). Use your answer to 3a to find an equation of the line tangent to the curve at the point $(1, -1)$.

3.1, 3.2, 4 (12 pts). Find the derivative of the given function and simplify your answers.

3.3, 3.5 a. $\dfrac{3x^7}{14} - \dfrac{4}{x^4} + 2x^3\sqrt[3]{x} - \sec x$ b. $\sin^{-1} x - \cos^{-1} x + e^{-1}$

3.3, 3.6 5 (17 pts). Find the derivative of the given function and simplify your answers.

a. $x\ln(\sec x)$ b. $(\sec x)^x$ c. $\ln\left(\dfrac{x}{x-2}\right)$

6 (15 pts). Find the derivative of the given function and simplify your answers.

3.6, 3.2 a. $\dfrac{1-\ln x}{1+\ln x}$. Express your answer without negative exponents or fractions-within-fractions.

3.4, 3.5 b. $\tan^{-1} x + \cot^{-1}(x^3)$. Express your answer in the form $\frac{a}{b} + \frac{c}{d}$. Simplify each fraction but do not add.

3.1, 7 (17 pts). Find the derivative of the given function and simplify your answers.

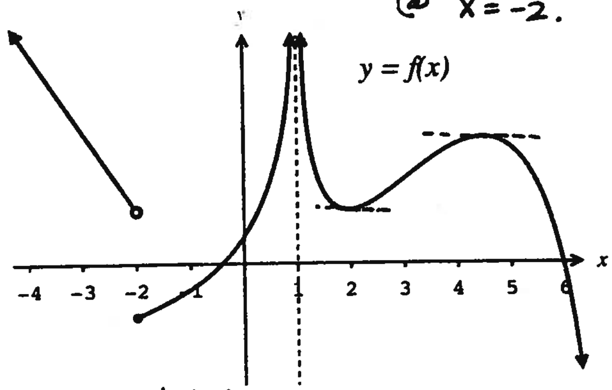
3.2, a. $\tan^2(x + \sin(3x))$

3.3,

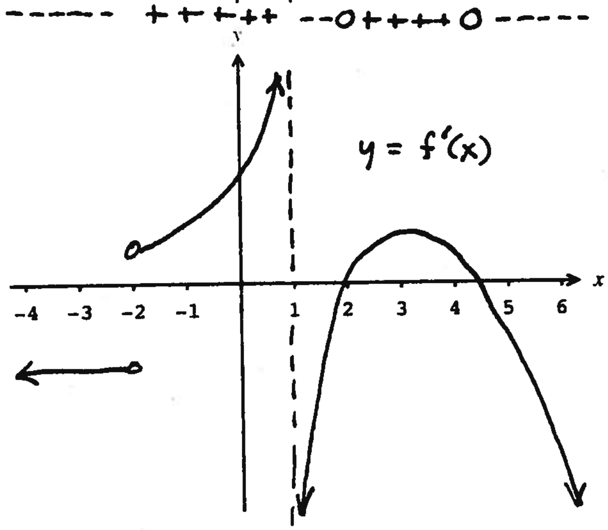
3.4 b. $(x+1)^5(3-x)^4$. Express your answer as a polynomial in factored form.

1a. $f'(1)$ DNE because $f(1)$ DNE
(2PT) $f'(-2)$ DNE because f discontinuous @ $x=-2$.

1b.
(10PT)



sign of f': – – – – – – + + + + + + – – 0 + + + + 0 – – – – –



2. $g(x) = xe^x$. Use product rule

$g'(x) = x'e^x + x(e^x)'$
$= 1e^x + xe^x$
$= (1+x)e^x$.

$g''(x) = (1+x)'e^x + (1+x)(e^x)'$
$= e^x + (1+x)e^x$
$= (1+1+x)e^x = (2+x)e^x$

3a. Differentiate w.r.t. x:

$2x + 2xy^2 + x^2 \cdot 2y\,y' = -2\sin(x+y)(1+y')$

Divide by 2, distribute on right side.

$x + xy^2 + x^2 y\,y' = -\sin(x+y) - y'\sin(x+y)$

$x + xy^2 + \sin(x+y) = -x^2 y y' - y'\sin(x+y)$
$= y'(-x^2 y - \sin(x+y))$

so $y' = \dfrac{x + xy^2 + \sin(x+y)}{-x^2 y - \sin(x+y)}$.

b. @ $(1,-1)$, $y' = \dfrac{+1+1+\sin(0)}{+1-\sin 0} = \dfrac{2}{1}$.

so line is
$y+1 = 2(x-1)$.

(8PTS)
4a. simplify before differentiating: $y = \dfrac{3x^7}{14} - 4x^{-4} + 2x^{3+\frac{1}{2}} - \sec x$.

(4PTS) $y' = \frac{3}{2}x^6 + 16x^{-5} + 7x^{5/2} - \sec x \tan x$. (Alt $(\csc x)' = -\csc x \cot x$)

4b. e^1 is a constant, so its derivative $=0$. $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-x^2}} - \dfrac{-1}{\sqrt{1-x^2}} + 0 = \dfrac{2}{\sqrt{1-x^2}}$

5a. $f' = \ln(\sec x) + x\dfrac{1}{\sec x} \cdot \sec x \tan x = \ln(\sec x) + x\tan x$.
(6 PTS)
(alt: $\ln(\csc x) - x\cot x$)

5b. $(\sec x)^x = e^{\ln(\sec x)^x} = e^{x \cdot \ln(\sec x)}$, so $f' = e^{x\ln(\sec x)}(\ln(\sec x) + x\tan x)$,
(from 5a)

alt. $f' = e^{x \cdot \ln(\csc x)}(\ln \csc x - x\cot x)$.

5c. $\ln\left(\dfrac{x}{x-2}\right) = \ln x - \ln(x-2)$, so $f' = \dfrac{1}{x} - \dfrac{1}{x-2}$. (deriv. of $x-2$ is 1)

(5PTS) or $\left(\ln\left(\dfrac{x}{x-2}\right)\right)' = \dfrac{1}{\frac{x}{x-2}} \cdot \dfrac{1(x-2) - x \cdot 1}{(x-2)^2} = \dfrac{-2}{\frac{x}{x-2} \cdot (x-2)^2} = \dfrac{-2}{x(x-2)}$.

(10 PTS)

6a. $\left(\frac{1-\ln x}{1+\ln x}\right)' = \frac{-\frac{1}{x}(1+\ln x) - (1-\ln x)\cdot\frac{1}{x}}{(1+\ln x)^2} \cdot \frac{x}{x} = \frac{-1-\ln x-1+\ln x}{x(1+\ln x)^2} = \frac{-2}{x(1+\ln x)^2}.$

(5 pts) alt: $\left(\frac{1+\ln x}{1-\ln x}\right)' = \frac{\frac{1}{x}(1-\ln x) - (1+\ln x)\frac{-1}{x}}{(1-\ln x)^2} = \frac{2}{x(1-\ln x)^2}.$

6b. $\frac{1}{1+x^2} + \frac{-1}{1+(x^3)^2}\cdot 3x^2 = \frac{1}{1+x^2} - \frac{3x^2}{1+x^6}.$

7a. $2\tan^1(x+\sin 3x)\cdot\sec^2(x+\sin 3x)(1+3\cos 3x)$

7b. $((x+1)^5(3-x)^4)' = 5(x+1)^4(3-x)^4 + (x+1)^5\cdot 4(3-x)^3(-1) = (x+1)^4(3-x)^3(5(3-x)-4(x+1))$
$= (x+1)^4(3-x)^3(11-9x).$

Alt: 5 4

derivative $= 4(x+1)^3(3-x)^5 + (x+1)^4\cdot 5(3-x)^4(-1) = (x+1)^3(3-x)^4(4(3-x)-5(x+1))$
$= (x+1)^3(3-x)^4(7-9x)$

(7a: 8 PTS
7b: 11 PTS)

Notes: 5b. power rule applies only when exponent is a constant.
5c, 6a. $\ln x$ is NOT $\frac{1}{x}$. $\frac{1}{x}$ is the derivative of $\ln x$
7a. Differentiate the squaring first, then the tangent, then $x+\sin 3x$.